Axiom disagreement and set-theoretic pluralism

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In recent years, the notion of *disagreement*, first described in the epistemological context by Elga (2007), has started to be applied in the context of various scientific disciplines (see Dellsén and Baghramian (2021) for a comprehensive account). Mathematics is not immune from disagreement between its practitioners, even though its nature might suggest the opposite (Jonas (2020) and Aberdein (2023)). Moreover, mathematics is subject to various forms of disagreement, as argued by De Toffoli and Fontanari (2023): axiom disagreement, philosophical disagreement, social-practice disagreement, heuristic disagreement, conception of proof disagreement, and putative proof disagreement. In her paper, De Toffoli focuses on putative proof disagreement, that is arguably the most present in general mathematical practice.

In this paper I will focus on axiom disagreement instead. According to De Toffoli and Fontanari (2023), axiom disagreement can arise regarding the axioms to add to our set-theoretic (or mathematical, or logical) system. For example, it is possible to disagree whether large cardinals axioms (i.e. axioms that state the existence of very large cardinals, like measurable cardinals) are a good addition to ZFC, or whether we should accept all instances of the law of excluded middle. To this kind of axiom disagreement I will also add disagreement on the preferred truth value of natural independent statements, i.e. statements with legitimate mathematical content that cannot be neither proved nor disproved by a certain theory (for example the Continuum Hypothesis CH, that's independent from ZFC). These two types of disagreement are deeply correlated: disagreement on the axioms to be adopted is usually rooted in disagreement on the preferred truth values of independent statements. For example, if the preferred solution to CH is that it is true, disagreement will arise with any set theorist that advocates for the adoption of the Proper Forcing Axiom (PFA), since this axioms implies the negation of CH (in particular, that $2^{\aleph_0} = \aleph_2$).

I argue that axiom disagreement, as described above, disappears if we adopt a pluralist stance regarding set-theoretic truth, while it cannot be avoided if we adopt a monist and universist stance. According to the universist, there exists only a unique, set-theoretic universe, while for the pluralism (see for example Hamkins (2012)) any model of any consistent set theory is a completely legitimate set-theoretic universe. I will argue that while the universist is stuck with axiom disagreement, since it has to define the One True Set Theory (i.e. a unique axiom system) that describes the unique set-theoretic universe, the pluralist can instead relativised any axiom choice to a particular set-theoretic universe in its "multiverse", thus avoiding any persistent disagreement.

To show that this is indeed the case in a precise manner, I will appeal to MacFarlane (2014) investigation of disagreement in the context of relativism. In particular, I will argue that in the pluralist case, we have that all the disagreement features (i.e. non-cotenability, preclusion of joint satisfaction, preclusion of joint accuracy, preclusion of joint reflexive accuracy) described by MacFarlane disappear, while in the case of universism they are all persistent.

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