Good Company, pluralism, and the foundations of mathematics

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In this paper, I plan to assess the abstraction principles that form the "good company" of the Cantor-Hume Principle (see Mancosu (2016)) according to their consequences for the foundations of mathematics. The main goal of Frege's logicist program was to reduce arithmetic to logic, thus founding mathematics on logic alone (Frege (1893-1903)). However, his use of Basic Law V and of the Cantor-Hume Principle led to Russell's Paradox. After Wright (1983) showed that there were ways to salvage Frege's program, there was a resurgence of interest on abstraction principles, culminating in the proof that in some cases abstraction principles can be used to derive the axioms of PA_2 (second order arithmetic) from impredicative second-order logic without falling into Russell's Paradox (Heck (2011)). Mancosu (2016) argued that there are several of these "good" abstraction principles, and that this poses the "Good Company Problem", that is, the problem of choosing which, among all these abstraction principles, is the "best" or "right" one. In the same book, Mancosu argued that they are mostly equivalent, since they can all be used to reduce arithmetic to logic. In this talk, I argue that this is not actually the case, if we assess them against the goal of developing a foundation of mathematics. In particular, I show that only the Cantor-Hume Principle can be actually used as the stepping stone for a foundation of mathematics, since it is the only one compatible with full ZF. The other ones all give rise to weaker fragments of set theory (General Set Theory, from Boolos and Thomson (1987), and "euclidean" set theories, see Parker (2013)), that cannot be used as a foundation of (classical) analysis or geometry (assuming the most minimal set of axioms). Finally, I point to the fact that a possible solution that saves all the "good" abstraction principles from the foundational perspective is domain pluralism, as sketched by Sereni et al. (2023).

References

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