The Free Choice Principle and the modal logic of forcing

Matteo de Ceglie

June 3, 2024

In this proposal, I plan to address whether the modal logic of forcing (as first developed by Hamkins and Löwe (2008)) satisfies the Free Choice Principle (Zimmermann (2000)). The Free Choice Principle (FCP) states that if a disjunction is possible, then each of the disjuncts is also possible: $\Diamond(\varphi \vee \psi) \to \Diamond\varphi \wedge \Diamond\psi$. Such a principle is usually not included as an axiom in any modal logic (Von Wright (1968)), since it would allow one to derive $\Diamond\psi$ from the single assumption $\Diamond\varphi$. The modal logic of forcing interprets $\Box$ and $\Diamond$ in terms of set-theoretic forcing: $\Box\varphi$ is true in M iff φ is true in all forcing extensions of M, while $\Diamond\varphi$ is true in M iff there exists at least one extension of M in which φ is true. In this set-theoretic context, the FCP means: if there is a forcing extension of M in which $\varphi \vee \psi$ is true, then there is a forcing extensions of M in which φ is true and another one in which ψ instead is true. A natural question is to ask whether the modal logic of forcing validates the FCP. In this proposal, I argue that, in the modal logic of forcing, the FCP actually holds, but only iff φ and ψ are independent *switches*, i.e. iff they are always possible in any set-theoretic model M. By contrast, the FCP doesn't hold for *buttons*, i.e. statements that can be forced true one time but then remain true in all further extensions. The philosophical upshot of this discussion is that it is possible to connect set-theoretic potentialism (i.e. the view that one can expand the set-theoretic universe by adding new sets) with the validity of the FCP. In particular, if the FCP holds, then the potentialist framework will be *linear* (i.e. new extensions includes all the previous ones), otherwise it will be *branching* (there are extensions with no relation to each other).

Keywords Free Choice Principle; Modal logic of forcing; Set-theoretic potentialism; Philosophy of Set Theory.

References

Hamkins, J., & Löwe, B. (2008). The modal logic of forcing. *Transactions of the American Mathematical Society*, *360*(4), 1793–1817.

Von Wright, G. H. (1968). An essay in deontic logic and the general theory of action. *Acta Philosophica Fennica*, (21).

Zimmermann, T. E. (2000). Free choice disjunction and epistemic possibility. *Natural language semantics*, *8*(4), 255–290.