What makes a mathematical proof "good"?
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In this paper, I plan to tackle the question of what makes a mathematical proof "good". Such a question has been in the centre of the philosophy of mathematics for a very long time. Recently, Granville (2023) investigated the question from the perspective of a working mathematician. In that paper, he distinguishes between "formal" and "culturally appropriate, intuitive" proofs. The first kind of proofs are the gap-less sequences of instances of axioms or application of rules that logicians know and love. According to Granville, these formal proofs are not "good" mathematical proofs, since they are not intuitive, and tend to muddle the reasons behind a mathematical result. On the other hand, we have "good" proofs. These are the intuitive proofs that we can (easily) understand, that convince us of the results they claim to prove, and that can be used as a springboard to prove novel and original theorems. To illustrate the difference, we can think of the proof of Pythagoras Theorem as explained in Euclid's *Elements*, or a corresponding derivation in Hilbert's axiomatic geometry of the same result. The first proof can be understood by any elementary student, while the second one needs much more mathematical maturity. Granville's claims are perfectly plausible and appealing, but he doesn't give any reason (other than some examples) on why formal proofs are not intuitive and culturally appropriate.

I argue that adherence to Grice's Cooperation Principle (Grice (1975)) provides those reasons. In other words: a formal proof is not a "good" mathematical proof because it doesn't satisfy Grice's maxims of the Cooperation Principle. According to Grice, our *utterances* should comply with a number of maxims that ensures the general cooperative nature of language. If an utterance doesn't comply with one, or more than one, of these maxims, then we usually search for a reason of such a decision, and go beyond the literal meaning of the utterance (these are what Grice calls *conversational implicature*). The classical example is answering the question "Can you pass me the salt?" with actually passing the salt, and not with a "Yes." (the implicature here being "If you can pass me the salt, can you please do that?").

More in detail, I propose that in the context of mathematical practice the Cooperation principle (CP) states:

CP Make your mathematical proof such as is required, at the stage at which it occurs, by the accepted purpose or direction of the research field in which you are engaged (Grice (1975))

and it is implemented in the following maxims (following the presentation from Korta and Perry (2020)):

- *Quantity*
  - Make your proof as informative as is required (for the current purposes of the research field).
  - Do not make your proof more informative than is required.

- *Quality*: try to make your proof one that is true.
  - Do not claim what you believe to be false.
  - Do not claim that for which you lack adequate evidence.

- *Relation*: be relevant.

- *Manner*: be perspicuous.
  - Avoid obscurity of expression.
  - Avoid ambiguity.
  - Be brief (avoid unnecessary prolixity).

- Be orderly.

- Frame whatever you say in the form most suitable for any reply that would be regarded as appropriate; or, facilitate in your form of expression the appropriate reply (added by Grice (1981)).

I contend that these maxims can be used to argue why a formal proof is not a "good" proof. In particular, given that both a formal proof and a "good" proof must satisfy the maxim of Quality (otherwise they wouldn't be proof in the first place), a formal proof doesn't satisfy any other maxim, while a proof, to be considered "good" should satisfy them.

References

Granville, A. (2023). Proof in the time of machines. *arXiv preprint arXiv:2305.02329.*

Grice, H. P. (1981). Presupposition and conversational implicature. *Radical pragmatics*, *183*, 41–58.

Grice, H. P. (1975). Logic and conversation. In *Speech acts* (pp. 41–58). Brill.

Korta, K., & Perry, J. (2020). Pragmatics. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Spring 2020). Metaphysics Research Lab, Stanford University.