

Mathematical practice and set-theoretic pluralism

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In this paper, I argue that set-theoretic pluralism – i.e. the position that there is no intended model of set theory, but a multiverse of them – is a better position than universalism – the position that there exists only one set-theoretic universe, namely the cumulative hierarchy V – regarding mathematical practice. To do so, I will focus on one particular part of mathematical practice: the correct formulation of the problem at hand. That this is a fundamental part of what the actual practice of mathematics entails can be already seen in Wittgenstein, that remarks in his Nachlass that “[...] the correct formulation of the question is already the answer.” (Wittgenstein Nachlass Ms-305,1[4]). Moreover, such a position can also be seen in the debate on the definiteness of CH (see e.g. Feferman (2014)), in which the interest of finding a solution to CH is challenged on the grounds that it is not a correctly formulated mathematical problem. Against this background, I argue that set-theoretic pluralism gives us the tools and methods to correctly formulate such problems. Following Hamkins (2012), the correct formulation of mathematical problems for pluralism involves exploring the multiverse, finding the conditions on which a statement can be made true or false with forcing or other set-theoretic methods. In particular, the CH can be seen, from the pluralist perspective, at least as a *definite* logical problem (in Feferman’s sense), if not even a definite ordinary mathematical problem.

References

- Feferman, S. (2014). “The Continuum Hypothesis is neither a definite mathematical problem or a definite logical problem”. Unpublished.
- Hamkins, J. (2012). “The Set-Theoretic Multiverse”. In: *Review of Symbolic Logic* 5.3, pp. 416–449.