V-logic Incompleteness

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Abstract

In this paper, I present a novel proof of the incompleteness of V-logic. V-logic is an infinitary logic first developed by Barwise, as $\mathfrak{M}$-logic, in the late '60, early '70. More work on the logic was done by Dickmann, Karp and Keisler, and in more recent times by Friedman and his research group on the Hyperuniverse. The language of this logic is the infinitary language $\mathcal{L}_{\kappa^+,\omega}$: in this language we admit conjunctions and disjunctions of length less than κ^+, the successor of the first strongly inaccessible cardinal. However we still admit only a finite block of quantifiers on front of these formulas (if we admit an infinite number them, then the result logic would be at the second order, as proved by Dana Scott). To this language we add a new constant, $\bar{V}$, to stand for the set theoretic universe V, and less than κ^+ costants, one for each object in V.

As proved by Barwise, if $\bar{V}$ is *countable*, then this logic is *complete*. Otherwise, V-logic is incomplete. In this paper, I present a novel proof of this fact, using set theoretic machinary (especially forcing). The interest of this fact (and proof) lies in the fact that V-logic is the centrepiece of current research on the set theoretic multiverse, and its completeness/incompleteness has important consequences for its development. Currently two multiverse conceptions (a multiverse conception is a pluralist framework for set theory that admits the existence of more than one set theoretic universe) are based upon V-logic: the Hyperuniverse (developed by Friedman and his research group) and the V-logic Multiverse (developed by Claudio Ternullo and the present author). The latter is particularly touched by V-logic incompleteness since it assumes an uncountable $\bar{V}$, thus is, apparently, based upon an incomplete logic. I will conclude by discussing some of the possible consequences of this fact and some strategies to overcome them.