A naturalistic justification of the Generic Multiverse with a core

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Structure of the Presentation

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Introduction outline

- Pluralism, Anti-Pluralism and Naturalism
- The reasons of the emergence of the multiverse:
  - Independent propositions;
  - Alternative set theories.
- A brief sketch of the main argument

Pluralism, Anti-Pluralism and Naturalism

Naturalism

Mathematical practice should be considered the final judge for questions in philosophy of mathematics.

Anti-Pluralism

There is only *one* set theoretic universe.

Pluralism

There are various set theoretic universes.

Independent propositions

Definition

A proposition ϕ is independent from a theory T if T neither proves nor refutes ϕ.

- the Axiom of Choice (AC);
- the Continuum Hypothesis (CH) and the Generalized Continuum Hypothesis (GCH);
- The Suslin Conjecture;
- Large Cardinals Hypothesis;
- The Axiom of Determinacy (AD).

Alternative set theories

- Non well-founded Set Theory (ZF^-);
- Constructible Set Theory ($ZFC + V = L$);
- Set Theory with Large Cardinals ($ZFC + LCs$);
- Set Theory with Determinacy ($ZF + AD$).

A brief Sketch of the argument

- The multiverse is just as good, when dealing with actual mathematical practice, as the single universe;
- Moreover, in the multiverse is possible to prove more things than in the single universe;
- Thus, from a naturalistic point of view, the multiverse should be preferred over the single universe.

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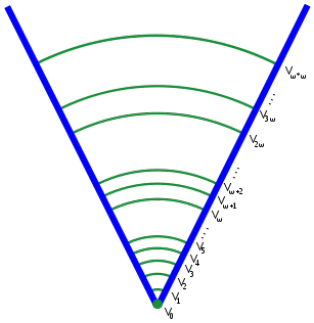
Background outline

- The classic set theoretic universe: the cumulative hierarchy
- The Multiverse conceptions in set theory
  - The broad multiverse;
  - The Generic Multiverse with a core (GM_H)
- The naturalistic approach

The classic set theoretic universe

- First of all, we consider only one set, the empty set $\emptyset$;
- Every new *finite* level is composed by all possible collections of sets formed in the previous levels, plus all the sets formed so far;
- The first *infinite* level, ω, is composed by all the sets formed before ω;
- We can extend this process in an *iterative* way.



Multiverse conceptions in set theory

The *broad* multiverse

All the possible universes are part of the multiverse, with no hierarchy nor criterion to sort them.

The generic multiverses

In this kind of multiverses we differentiate between universes using a strong logic (an idea owed to Woodin, from now on GM_Ω) or supposing the existence of a core (an idea owed to Steel, that is the GM_H).

The hyperverse

A multiverse of multiverses.

The broad multiverse

- There is no hierarchy nor criterion to sort the universes: every possible universe is part of the multiverse;
- A universe is a model of a possible set theory, a interpretation of the language of set theory;
- We can build even more universes applying set forcing to these universes, and we can then re-apply set forcing and so on.

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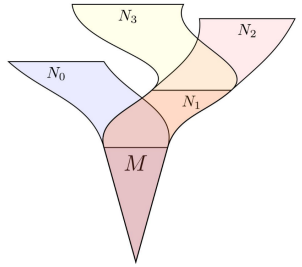
The Generic Multiverse with a core (GM_H)

Definition of the *core*

The core of the multiverse is the collection of all the statements that are true in *every* universe of the multiverse. We can then consider every universe of the generic multiverse and *extension* of its core.



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The Multiverse Language *MV* for GM_H

- The usual syntax of the language of set theory, but with two sorts:
  - sets (as usual);
  - *worlds*;
- this language is expressive enough to state versions of the axioms of *ZFC and* large cardinals hypothesis;

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The Axioms of the GM_H

Axioms

- *For each axiom ϕ of set theory and for every world W of the multiverse, there exists a translation of ϕ in W, denoted ϕ^W;*
- *Every world is a transitive proper class. An object is a set just in case it belongs to some world;*
- *If W is a world and $\mathbb{P} \in W$ is a poset, then there is a world of the form $W[G]$ where G is $\mathbb{P}$-generic over W;*
- *If U is a world, and $U = W[G]$, where G is $\mathbb{P}$-generic over W, then W is a world.*

The Axioms of the GM_H (cont.)

Axiom (Amalgamation)

If U and W are worlds, then there are G and H set generic over them such that $W[G] = U[H]$.

Axiom (Axiom H)

Every member of V can be defined in a model M in terms of a finite number of ordinals by a first order formula.

Consequences of Axiom H

- It implies that the multiverse has a core;
- can be used to study the definability of hierarchies;
- it is consistent with large cardinals hypothesis.

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The problem

Which is best for mathematical practice? The Single Universe framework or the Multiverse?.

UNIFY

Our framework should be *foundational*.

MAXIMIZE

The framework theory should prove the maximum number of isomorphisms.

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Some more about the naturalistic approach

- We could further refine UNIFY defining the following *foundationality* feature:
  - Meta-mathematical Corral;
  - Elucidation;
  - Shared Standard;
  - Risk Assessment.
- A candidate framework for mathematical practice should at least provide all these features.

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Structure of the Presentation

Main argument outline

- The foundationality of the GM_H
- Maximizing the descriptive power of the GM_H
- The non-foundationality of the other multiverse conceptions

The foundationality of the GM_H

- The multiverse core provides us all the foundationality feature needed to satisfy UNIFY:
  - Meta-mathematical Corrall;
  - Elucidation;
  - Shared Standard;
  - Risk Assessment.
- Thus, we can say that the GM_H and the Single Universe are just as good.
- Given this, there would be no reason to switch from the Single Universe to the GM_H.

Maximizing the power of the GM_H

- Lets consider two universes, A and A', that differ the least possible amount;
- Also, suppose that one of the universe, say A, is actually equal to V;
- We can say that A' differs from A for just one structure;
- In this case, we can prove that most of the structured of A' are isomorphic to some structures in A;
- All these new isomorphisms wouldn't be provable in a single universe case.
- We can easily see then how a multiverse can prove more isomorphisms than a single universe.
- Thus, considering MAXIMIZE, the GM_H is actually better than the Single Universe V.

Against the other multiverse conceptions

- All the other multiverse conceptions are equally powerful from the MAXIMIZE point of view;
- Although, they all fail the foundationality test:
  - The broad multiverse fails to provide Shared Standard and Risk Assessment;
  - Woodin's GM_Ω fails to provide us Meta-mathematical Corrall;
  - The hyperverse has the same problems of the broad multiverse, and moreover, cannot provide us with Elucidation.

Some details on why the broad multiverse doesn't work

- Every single universe is a different interpretation of the language of set theory;
- The metatheory is the most general possible, to accomodate all the various different universes;
- The main problem with this setting is that we have to redefine a different metatheory for every universe;
- Thus, we end up with a plurality of metatheory, in which we cannot define anything that is shared between all the universe.

Structure of the Presentation

Conclusions

- To conclude, we can say that the GM_H is our best candidate to be the framework for mathematical practice:
  - It is as foundational as the classic set theoretic framework;
  - Moreover, is the only multiverse conception that can claim to be foundational;
  - It proves more isomorphisms than the classical set theoretic framework;
  - Thus, from a naturalistic point of view, our only option is to accept that the GM_H is better for mathematical practice than the classical set theoretic framework.