Week 3

Worked-out example of checking a well-formed formula

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Consider the following string of symbols:

$$\neg(p \wedge \neg(\neg q \vee r)).$$

To check carefully if this string of symbols is a well-formed formula, we start searching for the *innermost* well-formed formula:

$$\neg(p \wedge \neg(\neg q \vee r))$$

and write it down in the linear construction table:

(1) "q" is a well-formed formula

After this first step, we proceed with the next symbol in the inductive definition. Remember that negation comes before disjunction:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

and we update our table:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula

Now we know that if we choose the disjunction the next formula ($\neg q\vee$) wouldn't be well-formed. Instead, we go for the next atomic formula:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

and we update our table:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula
(3) "r" is a well-formed formula

We can now go for the disjunction:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

that only now is a well-formed formula:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula
(3) "r" is a well-formed formula
(4) "$\neg q \vee r$" is a well-formed formula

The next step is to add the parenthesis:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

since a well-formed formula between parenthesis is still a well-formed formula:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula
(3) "r" is a well-formed formula
(4) "$\neg q \vee r$" is a well-formed formula
(5) "$(\neg q \vee r)$" is a well-formed formula

Like the second step, we now have a negation in front of a well-formed formula:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

and just as before this is also a well-formed formula:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula
(3) "r" is a well-formed formula
(4) "$\neg q \vee r$" is a well-formed formula
(5) "$(\neg q \vee r)$" is a well-formed formula
(6) "$\neg(\neg q \vee r)$" is a well-formed formula

Again, like the third step we cannot go for the conjunction now, since this would produce a non well-formed formula $(\wedge\neg(\neg q \vee r))$, so we need to search for the next possible symbol that would make a well-formed formula:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

and write it down:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula
(3) "r" is a well-formed formula
(4) "$\neg q \vee r$" is a well-formed formula
(5) "$(\neg q \vee r)$" is a well-formed formula
(6) "$\neg(\neg q \vee r)$" is a well-formed formula
(7) "p" is a well-formed formula

And now we can go for the conjunction:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

that only now is a well-formed formula:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula
(3) "r" is a well-formed formula
(4) "$\neg q \vee r$" is a well-formed formula
(5) "$(\neg q \vee r)$" is a well-formed formula
(6) "$\neg(\neg q \vee r)$" is a well-formed formula
(7) "p" is a well-formed formula
(8) "$p \wedge \neg(\neg q \vee r)$" is a well-formed formula

Just like the fifth step, we now add the parenthesis:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

and this also makes a well-formed formula:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula
(3) "r" is a well-formed formula
(4) "$\neg q \vee r$" is a well-formed formula
(5) "$(\neg q \vee r)$" is a well-formed formula
(6) "$\neg(\neg q \vee r)$" is a well-formed formula
(7) "p" is a well-formed formula
(8) "$p \wedge \neg(\neg q \vee r)$" is a well-formed formula
(9) "$(p \wedge \neg(\neg q \vee r))$" is a well-formed formula

And finally, in the last step we add the outmost negation:

$$\neg(p \wedge \neg(\neg q \vee r)),$$

that, since it is also a well-formed formula, concludes our check:

(1) "q" is a well-formed formula
(2) "$\neg q$" is a well-formed formula
(3) "r" is a well-formed formula
(4) "$\neg q \vee r$" is a well-formed formula
(5) "$(\neg q \vee r)$" is a well-formed formula
(6) "$\neg(\neg q \vee r)$" is a well-formed formula
(7) "p" is a well-formed formula
(8) "$p \wedge \neg(\neg q \vee r)$" is a well-formed formula
(9) "$(p \wedge \neg(\neg q \vee r))$" is a well-formed formula
(10) "$\neg(p \wedge \neg(\neg q \vee r))$" is a well-formed formula.