Week 2

Notes on the syntax of proposition logic

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One of the main goals of logic is the evaluation of arguments and lines of reasoning. The tools of logic are aimed not only at this evaluation, but also at the *simplification* of those arguments, to make them in turn easier to evaluate. The main device used in this simplification is the *symbolic language* (also known as *formal* language), i.e. the translation (formalisation) of the natural language to symbols. The main reason behind this move is that, just like with the arguments, it is easier to recognise patterns if they are presented in a symbolic fashion. Today we are going to introduce the symbolic language of *propositional* logic and its syntax, thus enabling you to manipulate formulas (sentences) and evaluate them.

The first thing we need for our symbolic language are *names*, i.e. some symbols to denote "things" we want to talk about (for example, "Socrates"). To this end, we use the letters from the start of the alphabet: $a, b, c, \ldots$. These letters denote a *particular* thing, for example Socrates, or a specific tree. If we don't know (or it doesn't matter) the specific thing we are talking about, we then use *variables*: $x, y, z, \ldots$. So, for example, in the following argument:

(1) Every man is mortal.
(2) Socrates is a man.

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(3) Socrates is mortal.

the names can be swapped like this:

(1) Every a is b.
(2) c is a.

---

(3) c is b.

where "a" is, *specifically* "man", "b is specifically "mortal" and "c" is specifically "Socrates". The second argument is thus the specific translation of the first one, and cannot stand for *any other* argument. If we want to make a more general translation, we can on the other hand use variables:

(1) Every x is y.
(2) z is x.

---

(3) z is y.

In this case we don't have a translation "one-to-one" between the variables and some things, but this argument can be translated both with "man", "mortal" and "Socrates" and with "dolphin", "immortal" and "John". We can even have and hybrid translation, for examples if we defined that $a, b, c, \ldots$ are *philosophers* names:

(1) Every x is y.
(2) a is x.

---

(3) a is y.

In this argument we can substitute whatever we want for x and y, but only philosophers names for a.

Now we have the symbols to talk *name* things, but we also want to say something about these things. In particular, we want symbols to say that a certain thing has a certain property. For this purpose, we use *capital letters*: $R, P, Q, \ldots$. For example, the property of "being red" can be symbolised like this: $R(a)$, i.e. "a is red". The same symbols are used to denote *relations*: for example, "Mary loves Jack" can be symbolised as follows: "$L(m, j)$". The *only* difference between properties and relations is that properties admit only *one* variable/name inside the parenthesis, while relations don't have an upper limit (but always at least one, obviously). For example, in the sentence "Michael thinks that Sarah likes Keith" the relation "x thinks that y loves z" can be expressed by the relation "$S(x, y, z)$", and we can substitute in it the names: $S(Michael, Sarah, Keith)$. As an example, consider the following argument:

(1) If Mary loves Robert, then she wants to marry him.
(2) Mary loves Robert.

---

(3) Mary wants to marry Robert.

can be translated as follows:

(1) If $L(m, r)$, then $M(m, r)$.
(2) $L(m, r)$.

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(3) $M(m, r)$.

with the translation key "$L(x, y)$" means "x loves y" and "$M(x, y)$" means "x marries y". Pay attention that relations *have an order*! Writing "$L(x, y)$" is *different* from writing "$L(y, x)$": "x loves y" is different from "y loves x".

At this point we can talk of virtually anything, but we still need the last, and most important, symbols for propositional logic. We need symbols to actually *connect* all these names, properties and relations. This is why we need *connectives*. These connectives stands for all those words we have used to connect and forms our sentences in the previous arguments:

- "$\neg$" stands for "not";
- "$\wedge$" stands for "and";
- "$\vee$" stands for "or";
- "$\rightarrow$" stands for "if ... then";
- "$\leftrightarrow$" stands for "if and only if".

These are the last symbols we need to translate any sentence and argument in the formal language. For example, the previous argument can be formalised like this:

(1) $L(m, r) \rightarrow M(m, r)$.
(2) $L(m, r)$.

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(3) $M(m, r)$.

Or we can translate a very complex sentence like "Mary marries Robert if and only if Mary loves Robert and not Jack, and Robert loves her and not Sybil":

$$M(m, r) \leftrightarrow L(m, r) \wedge \neg L(m, j) \wedge L(r, s) \wedge \neg L(r, s).$$

It is important to notice that we also need rules when using parenthesis. For example, the previous formula would be ambiguous, without the sentence in natural language. Lets consider a simpler example: $\neg x \wedge y$. What is the correct translation of this formula? Is it "not x and y" or "not x, and y"? To avoid such cases, we always put parenthesis when the interpretation of the formula could be ambiguous. For example, if what we really mean is "not x and y", we would formalise as "$\neg(x \wedge y)$", to avoid confusion with "not x, and y".