

# Week 1

## Notes on arguments

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Logic is interested in the study and evaluation of *arguments*. An argument is a set of sentences, the *premises*, that support and justify a *conclusion*. For example, the following text is not an argument:

Yesterday Jon went to take a coffee for breakfast. After doing that, he had a class at the department and an appointment with his dentist. He then returned home to take a nap.

On the other hand, the following is an argument:

If we don't manage to cut carbon emissions, then the climate will necessarily change.  
But if the climate will change, then Mankind will suffer. Thus if we don't manage to cut carbon emissions, then Mankind will suffer.

To help us evaluate these arguments, we enumerate every sentence in it and clearly separate the premises from the conclusion:

- (1) If we don't manage to cut carbon emissions, then the climate will necessarily change.
  - (2) But if the climate will change, then Mankind will suffer.
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- (3) Thus if we don't manage to cut carbon emissions, then Mankind will suffer.

Note that any argument can have an arbitrary number of premises, but always one conclusion. However, logic is not interested in any kind of argument. *Inductive* arguments, like this:

- (1) Today the sun rose.
  - (2) Yesterday the sun rose.
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  - (n) The sun rose every day last year.
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- (3) The sun always rises.

Such an argument is quite important for the empirical sciences, but it is not the one employed in logic. In logic we are interested in another kind of arguments, i.e. *deductive* arguments:

- (1) Every man is mortal.
  - (2) Socrates is a man.
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- (3) Thus, Socrates is mortal.

Such arguments are evaluated only *internally*, and we don't need to go see the world to make a decision on the "goodness" of them. Moreover, they are not subject to change if the status of the world change.

The main point of interest in the study of arguments is evaluating their *validity*, not if they are true (in the world) or false (in the world).

**Definition 1** (Validity of an argument). An argument is *valid* if and only if it is not the case that the premises are *accepted* and the conclusion is not.

“Accepting” a premise or a conclusion means assuming it is true (not only in our world, but in any possible world and situation we can think of). For examples, the following argument is valid, since we cannot accept the premises without accept also the conclusion:

- (1) If it is raining, then it is better to have an umbrella.
- (2) It is raining.

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- (3) Therefore, it is better to have an umbrella.

But this one is invalid, since it can be the case in which the premises are accepted *without* accepting the conclusion:

- (1) Some Greeks are philosophers.
- (2) Socrates is a philosopher.

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- (3) Socrates is Greek.

It should be stressed that the validity of an argument is totally disconnected from its truth (in the world). For example, the following example is a valid argument, although every sentence that composes it is false (in the world):

- (1) Every dolphin is immortal.
- (2) Socrates is a dolphin.

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- (3) Socrates is immortal.

Consequently we can have a valid argument in which all sentences are false in our world, one in which every premise is false and the conclusion is true, etc. etc. . In the special case in which the an argument is both valid and true, we say that the argument is *sound*:

**Definition 2** (Soundness of an argument). An argument is *sound* if and only if:

1. the argument is valid;
2. every sentence that makes the argument is true (in the world).

For example, the previous argument was valid, but not sound (in particular, every sentence was false in the world). Instead, the following argument is sound:

- (1) Every Greek is mortal.
- (2) Socrates is a Greek.

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- (3) Socrates is mortal.

All the valid arguments share a particular form, and are valid *because of that form*, and *not* because of their content. For example, the following two forms are *always* valid:

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| <ul style="list-style-type: none"><li>(1) Every A is B.</li><li>(2) X is A.</li><hr/><li>(3) X is B.</li></ul> | <ul style="list-style-type: none"><li>(1) If A then B.</li><li>(2) A.</li><hr/><li>(3) B</li></ul> |
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While the followings are always invalid (even if the content is actually true):

Another way to evaluate an argument is to search for contradiction in it. If two sentence in that argument are incompatible (we cannot accept both of them at the same time, because they are in contradiction), then the argument is *inconsistent*. The same is true for two (or more) arguments.

- (1) Some A is B.
- (2) X is A.

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- (3) X is B.

- (1) If A then B.
- (2) B.

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- (3) A

**Definition 3** (Consistency of an argument). An argument is *consistent* if and only if there are no contradictions in it, otherwise it is *inconsistent*. Two arguments are *consistent* if and only if their conclusions are not in contradiction, otherwise they are inconsistent.

For example, the following two arguments are both sound, but they are inconsistent:

- (1) Every man is mortal.
- (2) Socrates is a man.

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- (3) Socrates is mortal.

- (1) If a philosopher changes the world with his ideas, he is immortal.
- (2) Socrates has changed the world with his ideas.

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- (3) Socrates is immortal.