

# A naturalistic justification of the Generic Multiverse with a core

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# Structure of the Presentation

**1** Introduction

2 Background

3 The main argument

4 Conclusions

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2 Background

3 The main argument

4 Conclusions

# Introduction outline

- ▶ Pluralism, Universism and Naturalism
- ▶ The reasons of the emergence of the multiverse:
  - ▶ Independent propositions;
    - ▶ Alternative set theories.
- ▶ A brief sketch of the main argument

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  - ▶ Independent propositions;
  - ▶ Alternative set theories.
- ▶ A brief sketch of the main argument



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# Pluralism, Universism and Naturalism

## Naturalism

We should approach questions in philosophy of mathematics respecting how mathematics is actually practiced by mathematicians.

## Universism

There is only *one* set theoretic universe.

## Pluralism

There are various set theoretic universes.

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## A brief Sketch of the argument

- ▶ The multiverse is just as good, when dealing with actual mathematical practice, as the single universe;
- ▶ Moreover, in the multiverse is possible to prove more things than in the single universe;
- ▶ Thus, from a naturalistic point of view, the multiverse should be preferred over the single universe.

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2 Background

3 The main argument

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  - ▶ The broad multiverse;
  - ▶ The Generic Multiverse with a core ( $GM_H$ );
  - ▶ The Hyperuniverse
- ▶ The naturalistic approach

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# Multiverse conceptions in set theory

## The *broad* multiverse

All the possible universes are part of the multiverse, with no hierarchy nor criterion to sort them.

## The generic multiverses

In this kind of multiverses we differentiate between universes using a strong logic (an idea owed to Woodin, from now on  $GM_{\Omega}$ ) or supposing the existence of a core (an idea owed to Steel, that is the  $GM_H$ ).

## The Hyperuniverse

The collection of all countable transitive models of  $ZFC$ .



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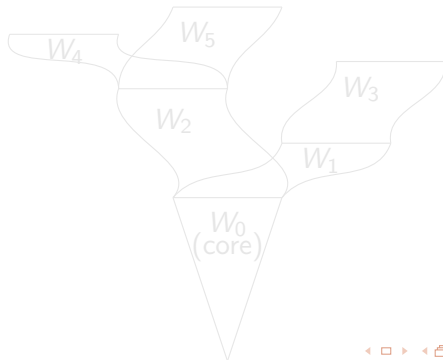
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# The Generic Multiverse with a core ( $GM_H$ )

## Definition of the *core*

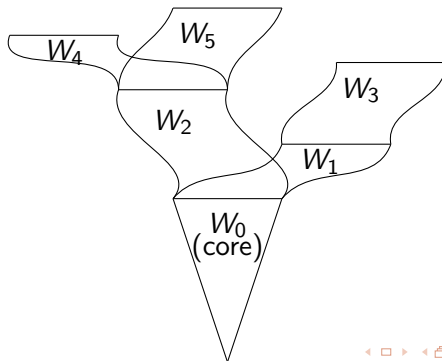
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# The Generic Multiverse with a core ( $GM_H$ )

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# General assumptions

## UNIFY

Our framework should be *foundational*.

## MAXIMIZE

The framework theory should be as powerful as possible, not restricting in any way the development of the foundations of mathematics (the framework theory should be the most Generous Arena for mathematics).

## Simple realism

We should take mathematical theories at face value: if mathematicians say that natural numbers exist, they exist.

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## Some more about the naturalistic approach

- ▶ We could further refine UNIFY defining the following *foundationality* features:
  - ▶ Meta-mathematical Corral;
  - ▶ Elucidation;
  - ▶ Shared Standard;
  - ▶ Risk Assessment.
- ▶ A candidate framework for mathematical practice should at least provide all these features.

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2 Background

3 The main argument

4 Conclusions

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## Against the other multiverse conceptions

- ▶ All the other multiverse conceptions are equally powerful from the MAXIMIZE point of view;
- ▶ Although, they all fail to satisfy our minimal assumptions:
  - ▶ The broad multiverse fails to provide Shared Standard and Risk Assessment;
  - ▶ Working multiverse fails to provide us with a mathematical formalism;
  - ▶ The hypothesis fails to solve problems of the level of the multiverse and the multiverse cannot be used as a justification of our world's reality and the multiverse is not a scientific hypothesis.

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# The fundamentality of the $GM_H$

- ▶ The multiverse core provides us all the fundamentality features needed to satisfy UNIFY:
  - ▶ Meta-mathematical Corral;
  - ▶ Elucidation;
  - ▶ Shared Standard;
  - ▶ Risk Assessment.
- ▶ Thus, we can say that the  $GM_H$  and the Single Universe are just as good.
- ▶ Given this, there would be no reason to switch from the Single Universe to the  $GM_H$ .



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## Maximizing the power of the $GM_H$

- ▶ Lets suppose that our multiverse is composed by only two universes: one is a model of  $ZFC$  and the other a model of  $ZF + AD$ ;
- ▶ In the multiverse, we retain all the results and true statements of  $ZFC$  and all the results of  $ZF + AD$ ;
- ▶ Moreover, we can also prove several more interesting isomorphisms in this simplified multiverse;
- ▶ On the other hand, in the Single Universe, we limit ourselves to only a subset of all the results we can prove in the multiverse;
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  - ▶ It is as foundational as the classic set theoretic framework;
  - ▶ Moreover, is the only multiverse conception that can claim to be foundational;
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