Introduction to Logic [296.617]

1st Midterm

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23 April 2020 / **Due:** 30 April 2020

1. For each of the following arguments, say if it is valid or not and, in each case, please justify your answer. [25 %]

   a)
   (1) If the Sun moves around the Earth, then Galileo was right.
   (2) The Sun moves around the Earth.
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   (3) Galileo was right.

   b)
   (1) Some men are immortal.
   (2) Plato is immortal.
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   (3) Plato is a man.

   c)
   (1) $A \to (B \vee C)$
   (2) A
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   (3) C

2. For each of the following sentences, use the truth-table method to find out whether it is contingent, a tautology, or a logical falsehood. [25 %]
   a) $\neg(\varphi \vee \psi) \to (\varphi \wedge \psi)$;
   b) $((\varphi \to \psi) \wedge \neg\psi) \to \neg\varphi$;
   c) $((\varphi \wedge \psi) \to \chi) \leftrightarrow (\neg\chi \wedge (\varphi \wedge \psi))$.

3. Translate the following sentences: [25 %]
   a) **From English to the language of propositional logic**: If John and Sally are Mark parent's, and Mark and Lucy are siblings, then either Mark and Lucy share both parents, or at least one of them.
   b) **From the language of propositional logic to English (please make use of the translation key provided)**: $(N(x) \wedge D(x, y)) \to \neg P(x)$
      - $P(x) :=$ x is a prime number;
      - $D(x, y) :=$ x is divisible by y;
      - $N(x) :=$ x is a natural number;

4. Prove the following formulas using Natural Deduction Rules for propositional logic (primitive rules only): [25 %]
   $$\vdash \chi \to (\psi \to (\varphi \to ((\varphi \wedge \psi) \wedge (\varphi \wedge \chi))))$$

Solutions

1. Arguments:
   a) Valid. The first premise is a conditional, while the second premise states that the antecedent is true. Then it cannot be that both premises are accepted and the conclusion is not.
   b) Invalid. There could be the case that Plato is a immortal but not a man.
   c) Invalid. Even if both premises are accepted, there is no entailment in accepting the conclusion (maybe is B that ought to be accepted, not C).

2. Semantic for propositional logic:
   a) Contingent:

φ	ψ	$\varphi \vee \psi$	$\neg(\varphi \vee \psi)$	$\varphi \wedge \psi$	$\neg(\varphi \vee \psi) \to (\varphi \wedge \psi)$
1	1	1	0	1	1
1	0	1	0	0	1
0	1	1	0	0	1
0	0	0	1	0	0

   b) Tautology:

φ	ψ	$\neg\varphi$	$\neg\psi$	$\varphi \to \psi$	$(\varphi \to \psi) \wedge \neg\psi$	$((\varphi \to \psi) \wedge \neg\psi) \to \neg\varphi$
1	1	0	0	1	0	1
1	0	0	1	0	0	1
0	1	1	0	1	0	1
0	0	1	1	1	1	1

   c) Contradiction:

φ	ψ	χ	$\neg\chi$	$\varphi \wedge \psi$	$(\varphi \wedge \psi) \to \chi$	$\neg\chi \wedge (\varphi \wedge \psi)$	$((\varphi \wedge \psi) \to \chi) \leftrightarrow (\neg\chi \wedge (\varphi \wedge \psi))$
1	1	1	0	1	1	0	0
1	1	0	1	1	0	1	0
1	0	1	0	0	1	0	0
1	0	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	0	1	0	1	0	0
0	0	1	0	0	1	0	0
0	0	0	1	0	1	0	0

3. Translations:
   a) $((F(J,M) \wedge M(S,M)) \wedge S(M,L)) \to (F(J,L) \vee M(S,L))$;
      i. $F(x,y) := x$ is the father of y;
      ii. $M(x,y) := x$ is the mother of y;
      iii. $S(x,y) := x$ and y are siblings.
   b) If x is a natural number and it is divisible by y, then x is not a prime number.

4. Natural Deduction for propositional logic:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi]^1 \quad [\psi]^2}{\varphi \wedge \psi}\langle\wedge\mathrm{I}\rangle \quad \dfrac{[\varphi]^1 \quad [\chi]^3}{\varphi \wedge \chi}\langle\wedge\mathrm{I}\rangle}{(\varphi \wedge \psi) \wedge (\varphi \wedge \chi)}\langle\wedge\mathrm{I}\rangle}{\varphi \to ((\varphi \wedge \psi) \wedge (\varphi \wedge \chi))}\langle\to\mathrm{I}\rangle}{\psi \to (\varphi \to ((\varphi \wedge \psi) \wedge (\varphi \wedge \chi)))}\langle\to\mathrm{I}\rangle}{\chi \to (\psi \to (\varphi \to ((\varphi \wedge \psi) \wedge (\varphi \wedge \chi))))}\langle\to\mathrm{I}\rangle$$