

#1 *NEW YORK TIMES* BESTSELLER

# LOGICOMIX



## AN EPIC SEARCH FOR TRUTH

APOSTOLOS DOXIADIS AND CHRISTOS H. PAPADIMITRIOU

ART BY ALECOS PAPADATOS AND ANNIE Di DONNA

B L O O M S B U R Y

# LOGICOMIX

APOSTOLOS DOXIADIS  
CHRISTOS H. PAPADIMITRIOU

*ART*

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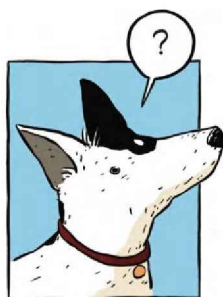
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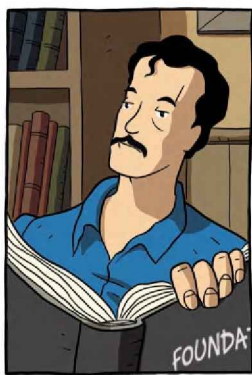
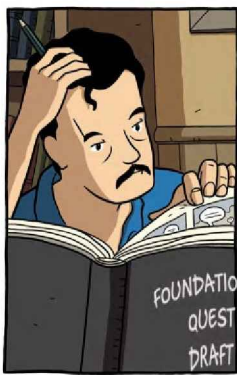
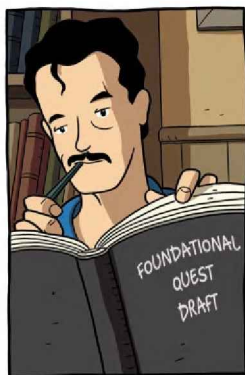
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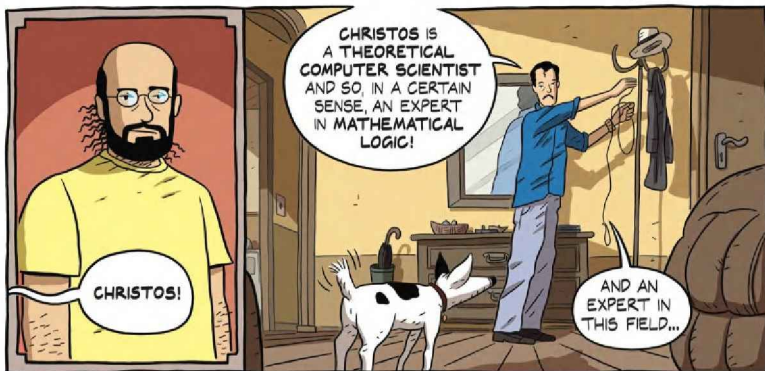
To our children,  
Eirene, Emma, Isabel, Io,  
Kimon, Konstantinos,  
Tatiana, Yorgos

*Υμῆς δ' ἔσεσθε πολλῶ κάρρονες.*

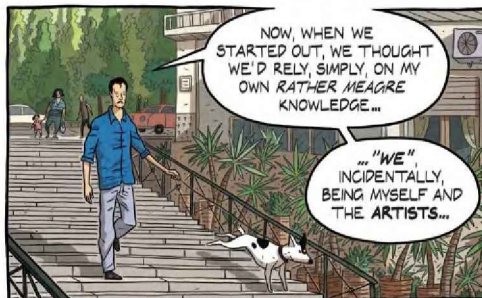
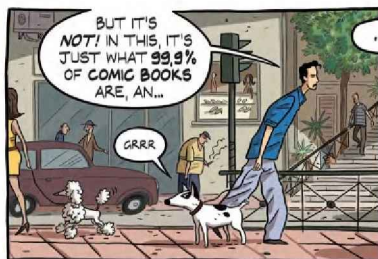
# OVERTURE















THE STORY BEGINS  
IN SEPTEMBER, 1939,  
AND EVEN MORE  
SPECIFICALLY—

THAT LATE?

YOU'LL  
SEE WHY? NOW,  
REMEMBER...

...SEPTEMBER 1<sup>ST</sup>,  
HITLER INVADIED  
POLAND.

THAT'S RIGHT.  
BLITZKRIEG!

...SO THE 'STUKAS' GUN  
DOWN THE INFANTRY  
AND CAVALRY.

...THE "PANZERS"  
RAM THROUGH  
THE DEFENCES.

THE COUNTDOWN  
TO A WORLD WAR  
HAS BEGUN...

AS RESISTANCE CRUMBLES BEFORE A SUPERIOR FORCE, NAZI PROPAGANDA TRIES TO TURN THE POLISH PEOPLE AGAINST THEIR NATURAL ALLY.



**This is your work, Britain!**

THE TASK IS MADE EASIER BY THE UNITED KINGDOM'S HAVING SIGNED THE "MUNICH PACT" OF NON-AGGRESSION WITH HITLER.

NOW LISTEN...

...THREE DAYS AFTER THE INVASION, BERTRAND RUSSELL, THEN MORE WIDELY KNOWN AS A PUBLIC THINKER, IS SCHEDULED TO GIVE A TALK AT AN AMERICAN UNIVERSITY ON THE "ROLE OF LOGIC IN HUMAN AFFAIRS".

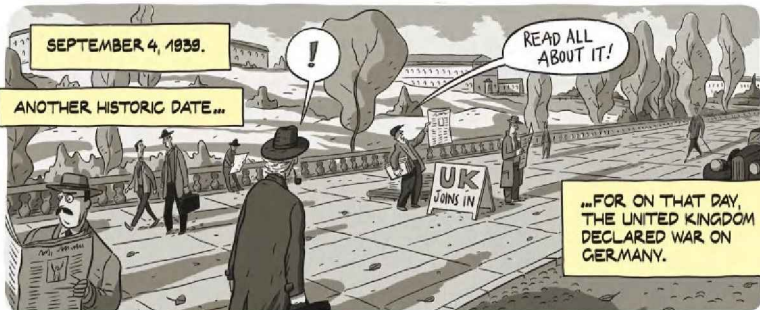


EXTRA!!!  
EXTRA!!!

AND IT'S ON THAT DAY THAT OUR STORY BEGINS...

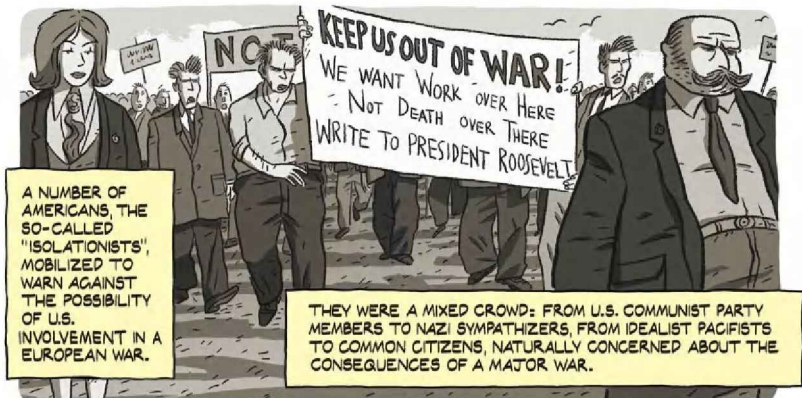
SEPTEMBER 4, 1939.

ANOTHER HISTORIC DATE...



READ ALL ABOUT IT!

...FOR ON THAT DAY, THE UNITED KINGDOM DECLARED WAR ON GERMANY.













...DO YOU  
EVER MISS  
ATHENS?

HOW  
COULD I NOT?  
IT'S WHERE  
I GREW UP...

...THOUGH  
WHERE I LIVE IS  
NO HELL-HOLE  
EITHER!





YOU DO WELL TO START WITH **RUSSELL**: HE IS ONE OF THE STORY'S BIG STARS!

BUT THAT'S NOT THE ONLY REASON WE CHOSE HIM. HE ALSO HAS THESE **OTHER SIDES...**

AH YES... POLITICAL ACTIVIST, PHILOSOPHER, LADIES' MAN!

BUT MORE: HIS **COMPLEXITY** AS A CHARACTER!

YOU KNOW, THE "FOUNDATIONAL QUEST" IS ESPECIALLY FASCINATING TO ME, FOR MY WORK. MOST OF THE GREAT IDEAS IN MY FIELD HAVE THEIR SEED IN **THERE** SOMEWHERE!

WELL, WE FOCUS ON THE **PEOPLE!**

THEIR IDEAS INTEREST US ONLY TO THE EXTENT THAT THEY SPRING FROM THEIR **PASSIONS.**

BY THE WAY, I READ GIAN-CARLO ROTA'S ARTICLE YOU SENT, ON THE CURIOUSLY HIGH RATE OF PSYCHOSIS IN THE LIVES OF THE FOUNDERS OF LOGIC.

SO, WHAT'S THE ALTERNATIVE? "THEY BECAME LOGICIANS FROM MADNESS?"

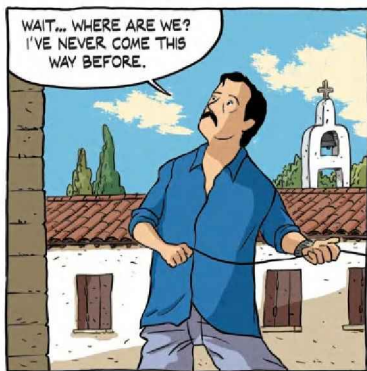
DON'T! DON'T YOU REVEAL "WHODUNIT" YET!

DOESN'T THAT MAKE YOU THINK? ESPECIALLY SINCE, CONTRARY TO POPULAR LEGEND, MOST OTHER MATHEMATICIANS ARE NOT MAD!

THAT'S CLOSER TO THE TRUTH, IF YOU REPHRASE IT AS --

SO, WHY SUCH A HIGH RATE OF MADNESS AMONG LOGICIANS, PARTICULARLY? MIND YOU...

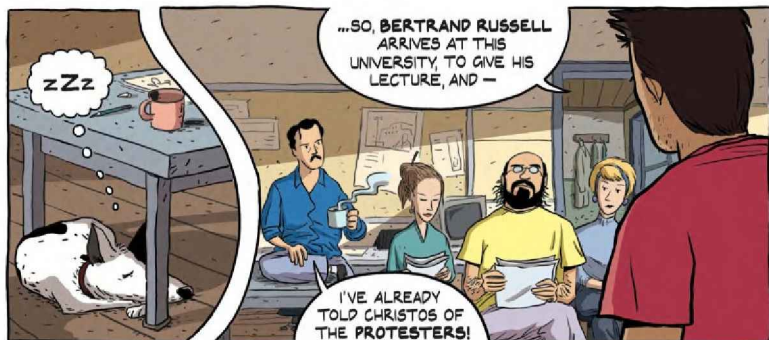
...I THINK THE CLICHE "THEY WENT MAD FROM TOO MUCH LOGIC" WON'T HOLD WATER!



\* Apostolos' dog is not named after Japanese comics. "Manga" is a slang word in Greek, meaning something like "cool dude" (U.S.) or "Jack-the-lad" (U.K.).



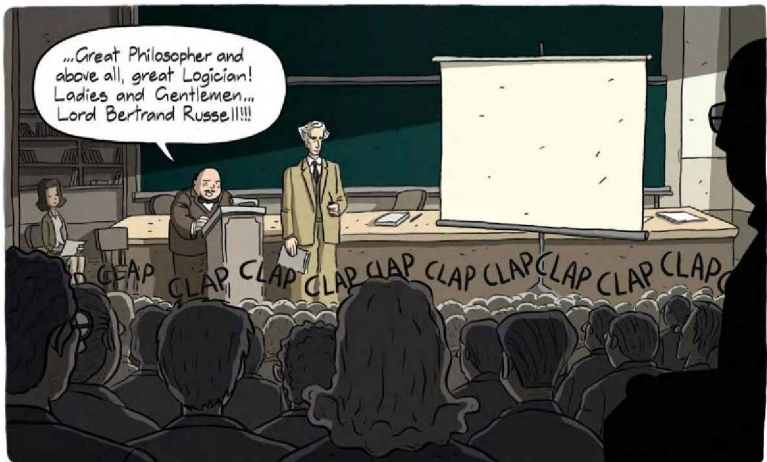
\* Annie is French.

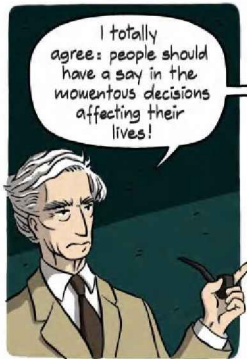
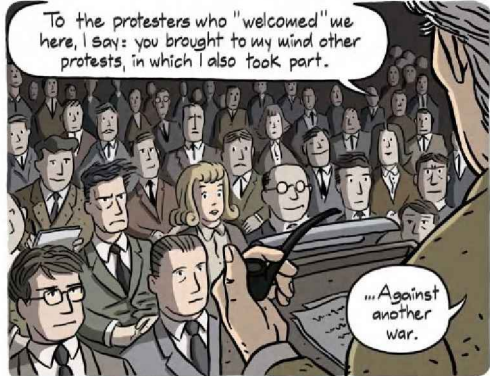


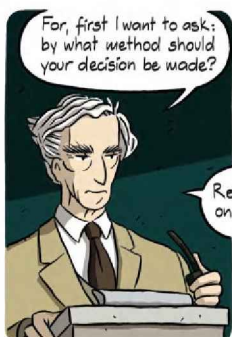
# 1. PEMBROKE LODGE







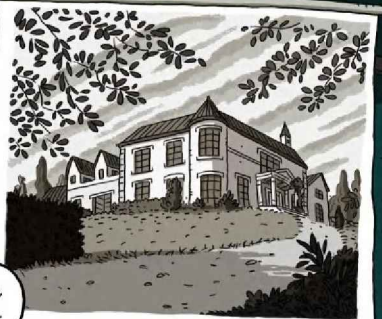




I was a young child, and thus by no means yet a logician, when I arrived at "Pembroke Lodge".



Here it is.



...This was the home of my paternal Grandfather, Lord John Russell.



And of my Grandmother, whom everyone called "Lady John"!



The day of my arrival is my first clear memory.

It was raining.



MIND YOU DON'T SOIL YOUR SHOES, MASTER BERTIE!

And the umbrellas were black.

My Grandfather was an important man. In fact, he had been Prime Minister of Great Britain.

HELLO, CHAPPIE! SEE THE COIN? I'LL MAKE IT DISAPPEAR!

NOT NOW, JOHN!

...But he was not Prime Minister of Pembroke Lodge!

FOLLOW ME, YOUNG MAN!

"Lady John" led me into a maze of corridors, up and down stairs, through countless doors...

All the while expounding on the rules that would govern my new existence...

YOU SHALL BE WELL-GROOMED AT ALL TIMES...

YOUR CLOTHES SHALL ALWAYS BE TIDY...

...YOUR HAIR PROPERLY BRUSHED!

YOU MUST NOT RAISE YOUR VOICE, FOR ANY REASON...

YOU SHALL NOT TALK BACK...

YOU SHALL NEVER GO BAREFOOT...

Rules, rules, rules...

LIVING ROOM

DINING ROOM

SMOKING ROOM

LAUNDRY

COATS

(50)

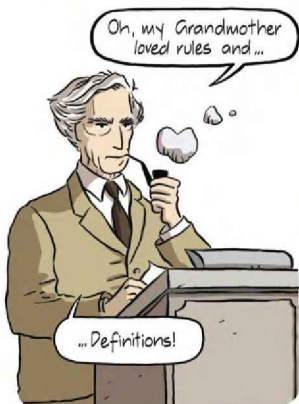
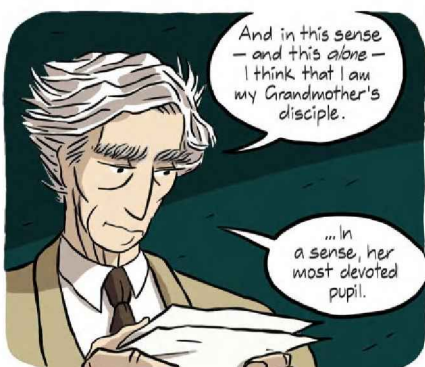
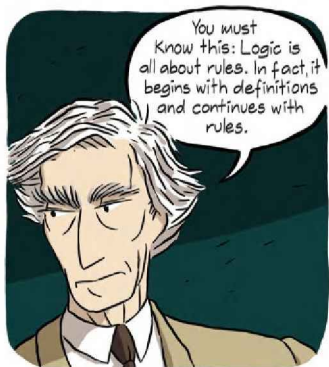
DM

SCHOOL ROOM

KITCHEN

PANTRY

PLAYING YARD



TU  
WOO

In my new abode,  
the hooting of the owl  
resounded with a new  
ominousness.



The wind sighing through  
the trees echoed my own  
feelings.

WHOOOSH

...And then, suddenly,  
I heard an unearthly  
moan!

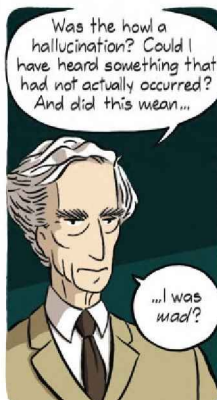


Although it contained  
all the unbridled emotion  
of the animal, the moan  
was, in an uncanny sort  
of way, human.





I was in for a surprise.



Thankfully, my curiosity did not let me ruminate too much...



IT CAME FROM UP THERE!

...But drove me to investigate.



As on many future occasions, my eagerness to know was stronger than my fear.



THAT'S IT!



THE LORD PRESERVE ME...



LOCKED!



Despite its heavy share of restrictions, it was only in the old mansion's garden that I acquired my first, rare experiences of freedom.



In it, there was always something interesting to do.





Oh, what a treasure  
trove this was!

HAVE YOU  
**READ** ALL  
THESE BOOKS,  
GRANDFATHER?

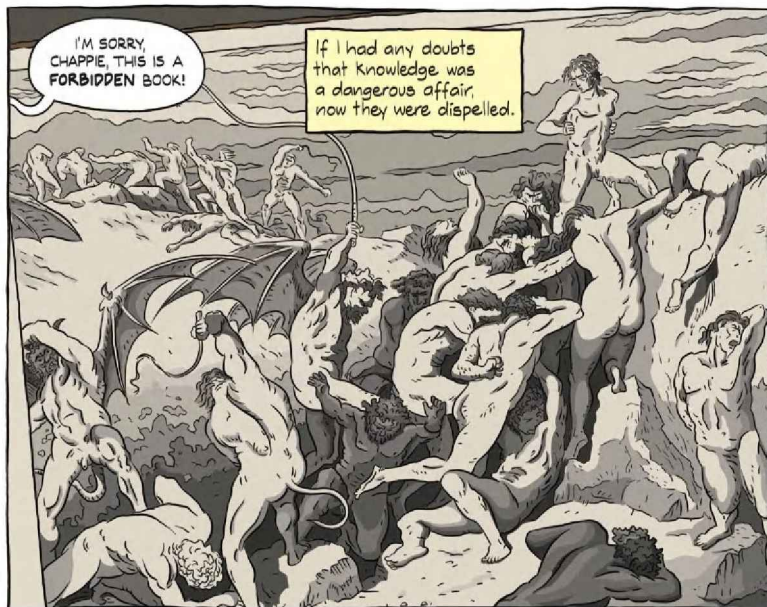


MOST  
BELONGED TO  
MY GRANDFATHER!



YET,  
I ENJOY BEING  
IN THE MIDST OF  
GREAT IDEAS!  
HE, HE!

DANTE  
*Inferno*



I'M SORRY,  
CHAPPIE, THIS IS A  
FORBIDDEN BOOK!

If I had any doubts  
that knowledge was  
a dangerous affair,  
now they were dispelled.



"FORBIDDEN"  
BOOK?

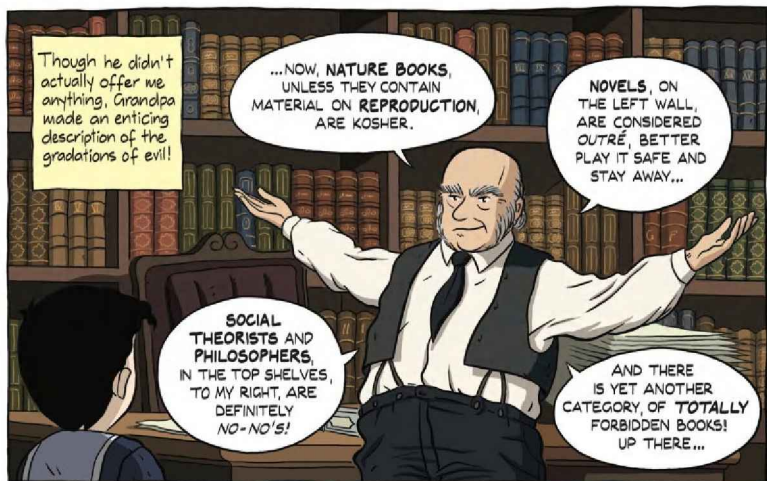


YOUR  
GRANDMOTHER  
IS A GREAT BELIEVER  
IN NOT EATING OF  
THE FRUIT OF  
KNOWLEDGE!

AND  
I HAVE TO  
PLAY SOME  
KIND OF  
CERBERUS...

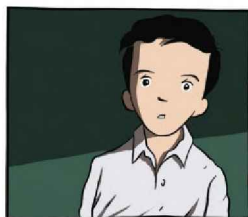


But, actually,  
I think the role of  
the serpent was  
more appropriate!









The next day, I paid my last respects to Grandfather.



Sitting beside him, I couldn't help thinking that Grandmother's reaction was more anger than grief...



...An anger whose cause I thought I knew.



When the storm came, that evening...

CRACK

...My very worst fears  
were deafeningly  
confirmed.

Grandfather had been  
punished, for giving  
us of the Fruit of  
the Tree of Knowledge.

Now I was being warned,  
and warned most clearly  
indeed:

"Do not repeat  
his folly!"

If I dared venture on another expedition in forbidden territory...

... An avenging deity's thunder would strike my world flat!

1879

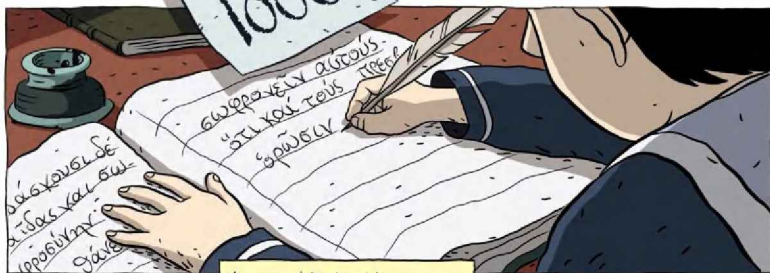
Article

Being who she was, Grandmother could not trust my education to a school.

Διδάσκουσι δὲ τοὺς παῖδας καὶ σωφροσύνην μέγα δὲ συμβάλλεται εἰς τὸ μανθάνειν σωφρονεῖν αὐτοὺς...

So she hired a tutor.

1880



An expert in dead languages...



δε τοὺς ἄλλους omni omni

NOW YOU SAY IT!

ommes  
omniium  
omniibus  
ommes  
omniibus  
ommes

But no outsider could be trusted with my religious education.



"THE HAND OF THE LORD WAS UPON ME, AND CARRIED ME OUT, AND SET ME DOWN IN THE MID OF THE VALLEY..."

IN THE MIDST OF THE VALLEY, BOY!



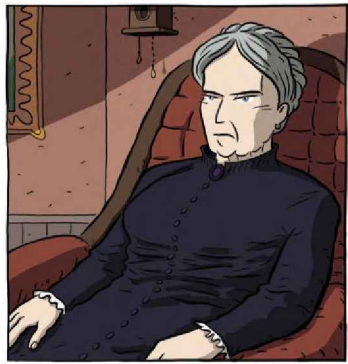
"...IN THE MIDST OF THE VALLEY WHICH WAS FULL OF BONES. AND THEY WERE VERY DRY..."



GRANDMOTHER?



PLEASE TELL ME WHERE MY PARENTS ARE!



WELL, LET US SAY THEY ARE OUT OF HARM'S WAY.



Yet, as I grew, my situation considerably improved.



One spring morning...

BERTRAND?



...Brought me pleasant surprises!

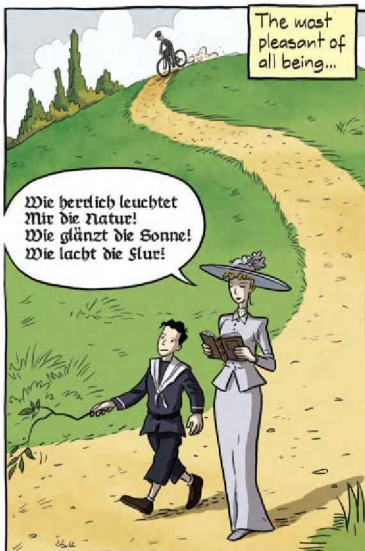


I AM YOUR NEW...

... GERMAN TEACHER!



The most pleasant of all being...



Wie herrlich leuchtet  
Mir die Natur!  
Wie glänzt die Sonne!  
Wie lacht die Stur!

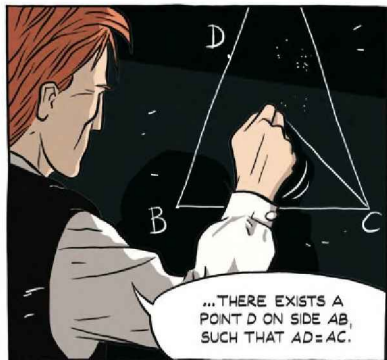
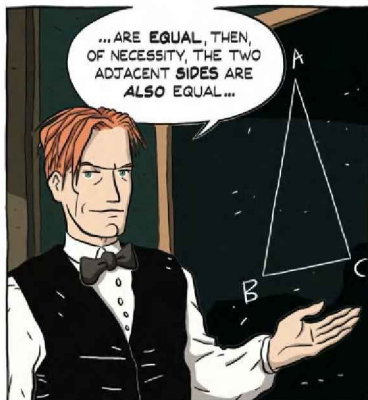
A young man...

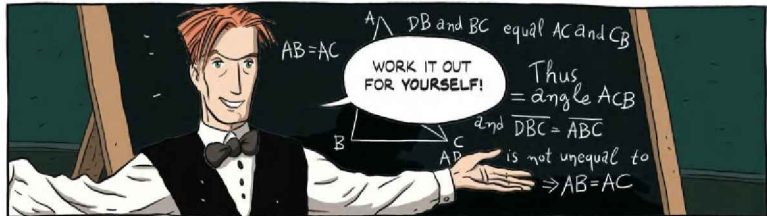


...Whose beliefs somehow had escaped Grandmother's stern controls!



It was he who introduced me to a very old gentleman.











And then it came: as I slept one night, I received an unusual message.



I can only surmise the identity of the messenger...



...But I am totally certain of its effect on me.

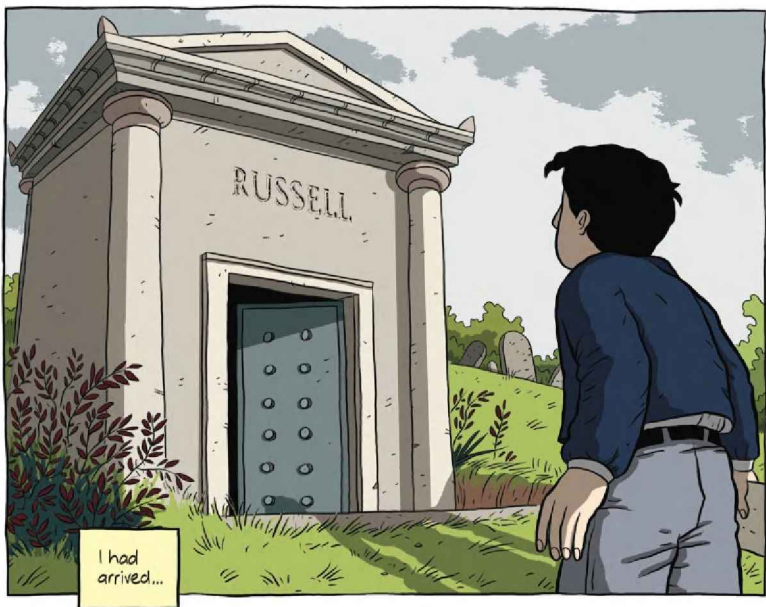
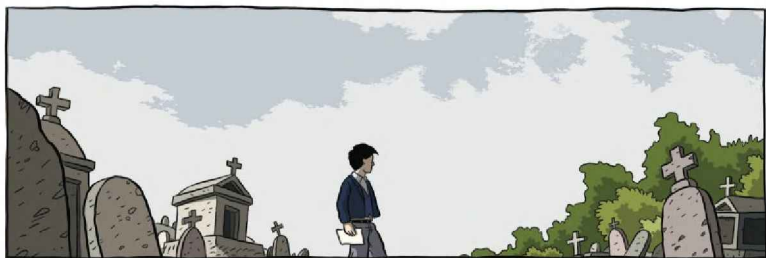


That same day, I escaped Grandmother's attention, to follow the route indicated in the mysterious note.



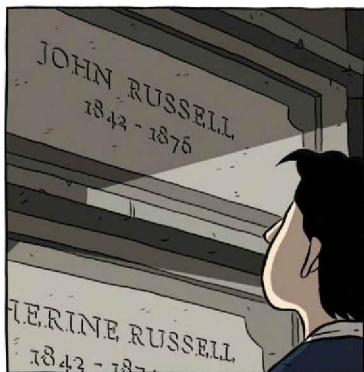
My curiosity barely exceeded the fear of getting caught.





I had arrived...











At long last, I could put faces to my family.



Rachel Russell,  
my sister.

Her death from diphtheria, just after she turned six, sparked-off the chain...

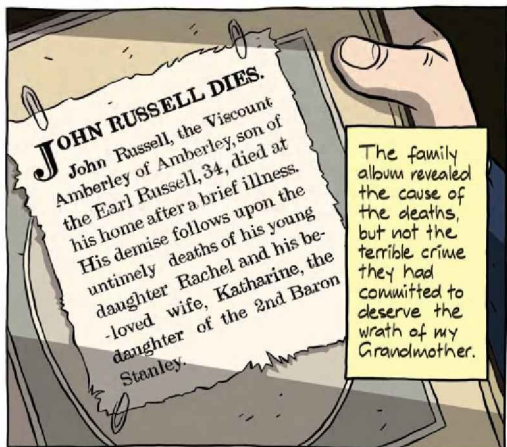


Katharine Russell,  
*née* Stanley...

...My mother died of the same disease...



...Their deaths killing my father, who had lost all will to live..



### JOHN RUSSELL DIES.

John Russell, the Viscount Amberley of Amberley, son of the Earl Russell, 34, died at his home after a brief illness. His demise follows upon the untimely deaths of his young daughter Rachel and his beloved wife, Katharine, the daughter of the 2nd Baron Stanley.

The family album revealed the cause of the deaths, but not the terrible crime they had committed to deserve the wrath of my Grandmother.



Who, always keen to explain...

...Barged into the scene of the crime.



WHAT DO YOU THINK YOU ARE DOING, YOUNG MAN?

But I was too shocked to be intimidated.



RESPECTFULLY GRANDMA, I KNOW WHAT I'M DOING!



I'M TRYING TO FIND OUT EVERYTHING THAT YOU KEPT FROM ME!

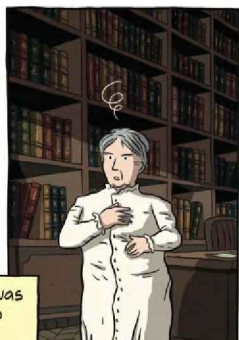


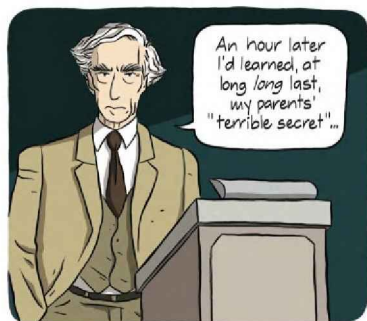
WHY... I SAY...



AND THAT'S PRECISELY WHAT I INTEND TO DO!!!

Grandmother was too shocked to react.





...Which was nothing more than that they'd lived in a ménage à trois with a sickly young man.



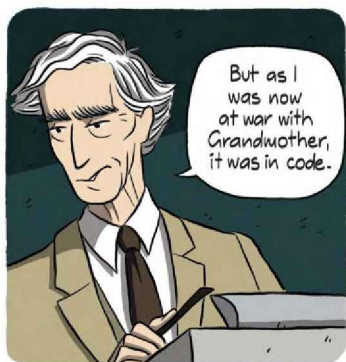
But there were also hints of a darker truth...

The late John Russell is survived by his parents Lord John Russell and Lady Frances Anna Maria Elliot Russell, his elder brother, William, and his young son Bertrand Arthur William Russell, who has been taken into the care of his Grandparents.

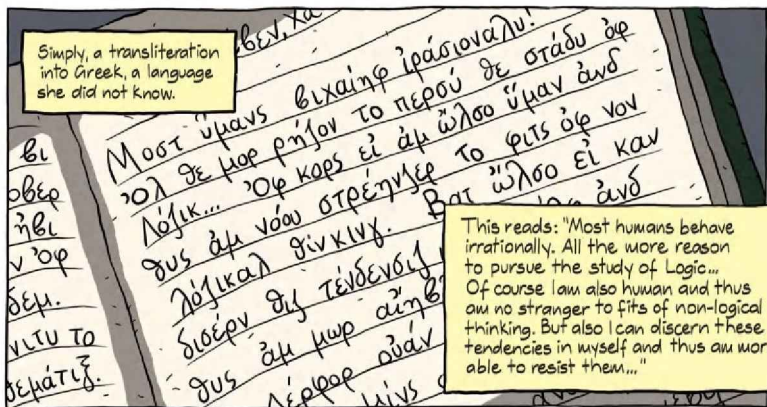
I HAVE AN UNCLE?



That year,  
I began my first  
philosophical work:  
an intimate diary!



But as I  
was now  
at war with  
Grandmother,  
it was in code.



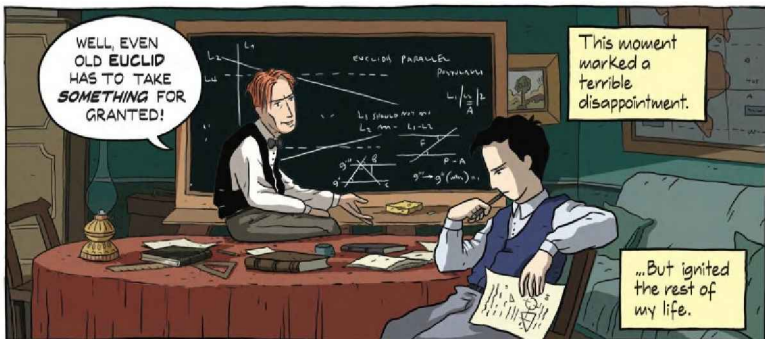
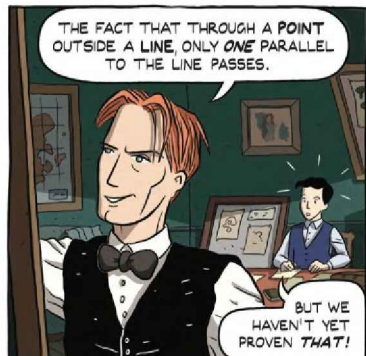
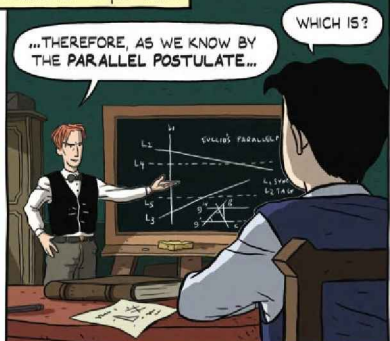
Simply, a transliteration  
into Greek, a language  
she did not know.

This reads: "Most humans behave  
irrationally. All the more reason  
to pursue the study of Logic...  
Of course law also human and thus  
am no stranger to fits of non-logical  
thinking. But also can discern these  
tendencies in myself and thus am  
able to resist them..."




"Greek Exercises",  
as my notebook  
was called for  
reasons of  
subterfuge,  
became a haven  
for all my secret,  
forbidden  
thoughts.

I had a lot to put in it!









...FOR THEIRS IS THE  
KINGDOM OF HEAVEN.  
BLESSED ARE THEY THAT  
MOURN, FOR THEY SHALL  
BE COMFORTED.  
BLESSED ARE THE...

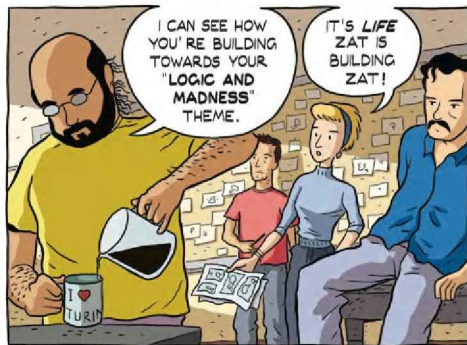
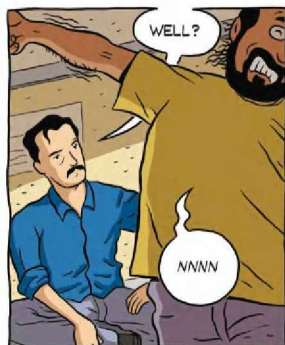
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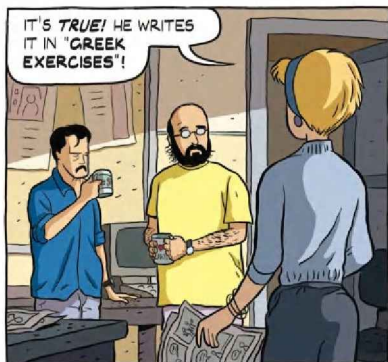
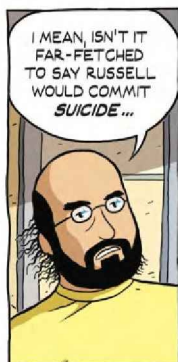
Here then was  
my uncle, my  
father's brother.



## 2. THE SORCERER'S APPRENTICE

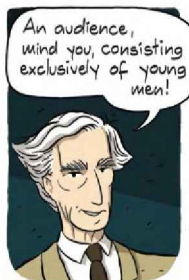


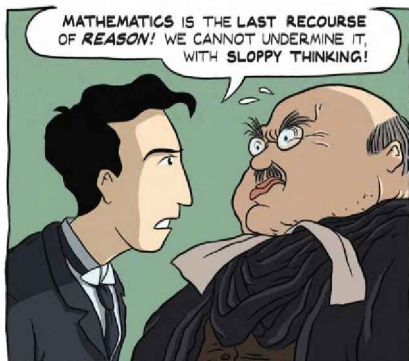
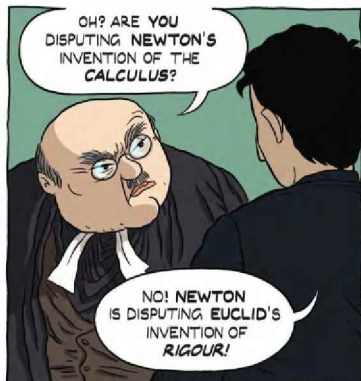
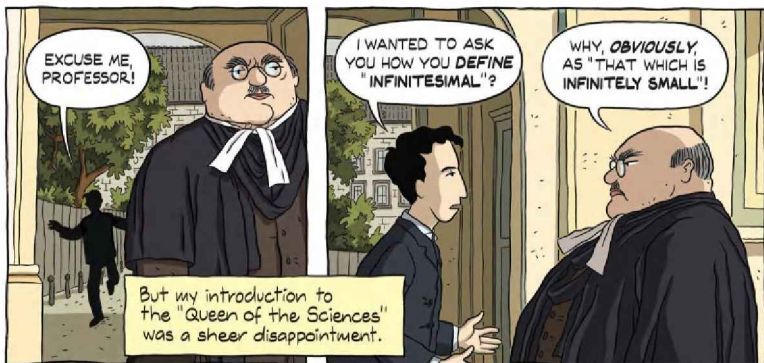




\* "I would kill myself."

\*\* "Yes! And now, if you don't mind, the story continues..."



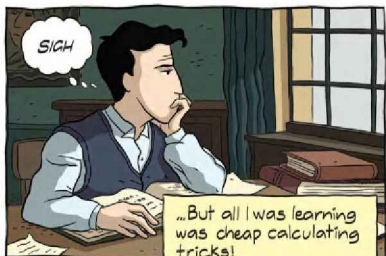




Studying Mathematics I had hoped to penetrate the essence of truth...



SIGH



...But all I was learning was cheap calculating tricks!

Still, my thirst for knowledge did not diminish.



EXCUSE ME, WHICH WAY TO THE UNIVERSITY LIBRARY?

At Cambridge, I discovered new worlds.



New options.

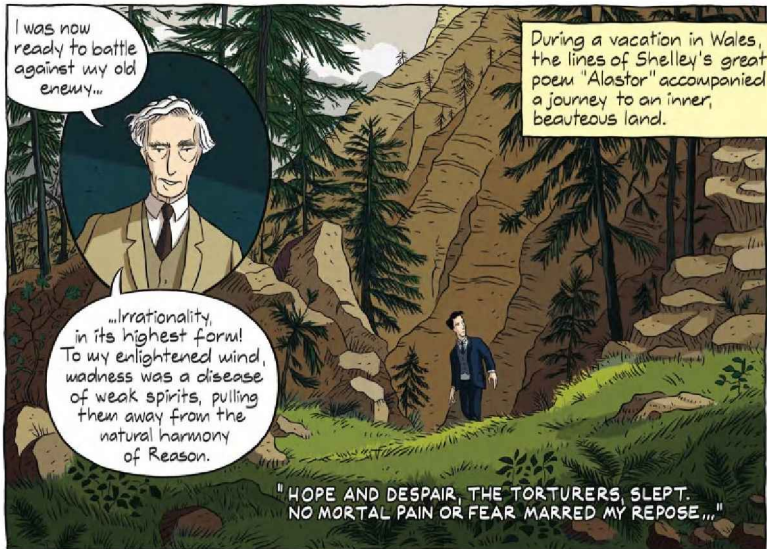


"THERE ARE NO PRINCIPLES' BAZAROV SAID. 'I LIKE TO DENY! MY BRAIN IS MADE ON THAT PLAN!'"



The deliciously wicked new novels helped me gradually lose my fear of Grandmother's stern commands.



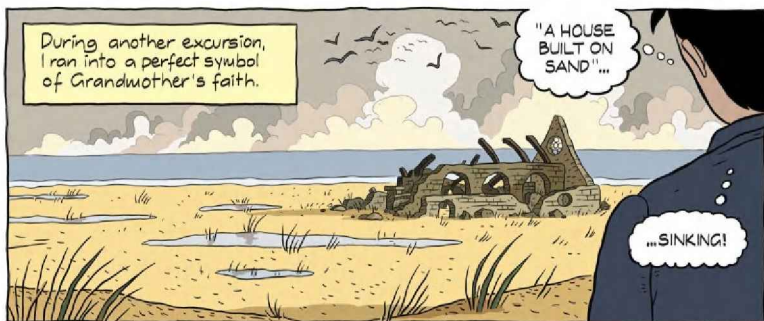




" EARTH, OCEAN, AIR,  
BELOVED BROTHERHOOD!  
MOTHER OF THIS UNFATHOMABLE WORLD!  
FAVOUR MY SOLEMN SONG, FOR I HAVE  
LOVED THEE EVER AND THEE ONLY!!! "

I was strong  
enough to  
cry out.

At last, I could  
turn my back  
on my dark  
legacy.



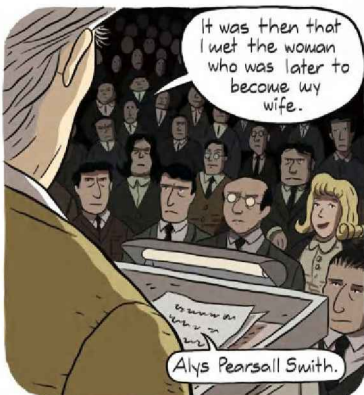
In those years, I was often accompanied by extreme inner tension.



My near-maniac passion for certain, absolute knowledge...

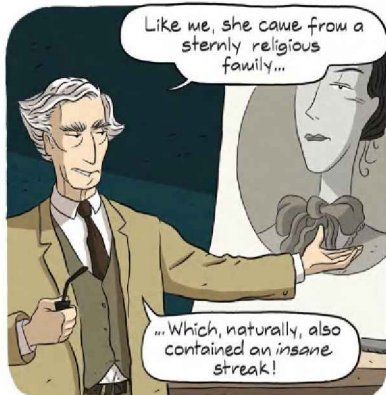


... Was doubtless made more intense by loneliness.



It was then that I met the woman who was later to become my wife.

Alys Pearsall Smith.



Like me, she came from a sternly religious family...

... Which, naturally, also contained an insane streak!

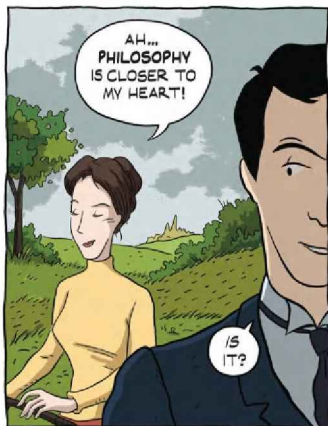
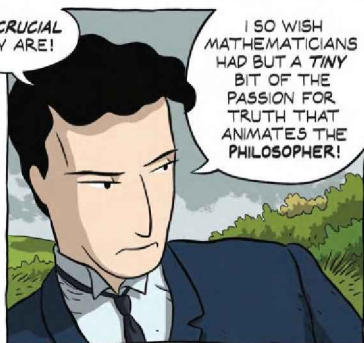
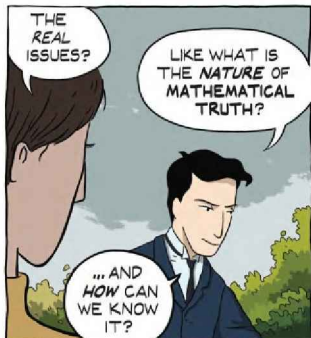
Yet our friendship at first progressed sanely...



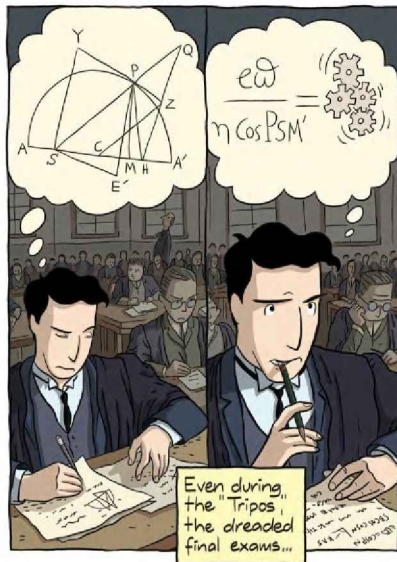
...In fact, a bit rather *too* sanely for my taste!

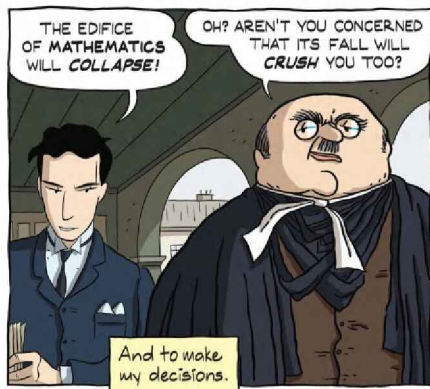
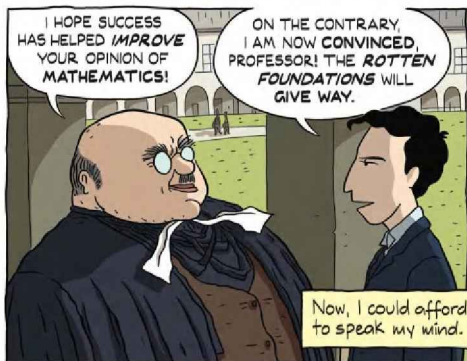
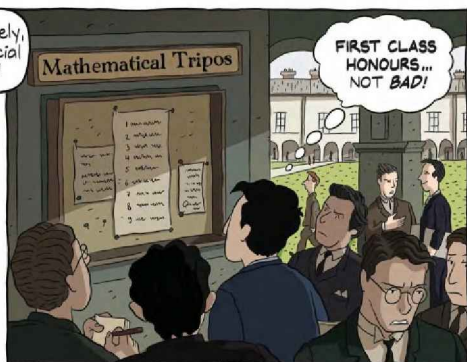
"PROFESSIONAL MATHEMATICIAN"? WHAT DOES A PROFESSIONAL MATHEMATICIAN DO?  
LONG SUMS?

YOU ARE NOT FAR OFF THE MARK THERE!

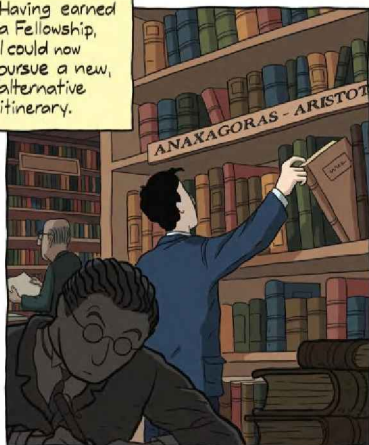




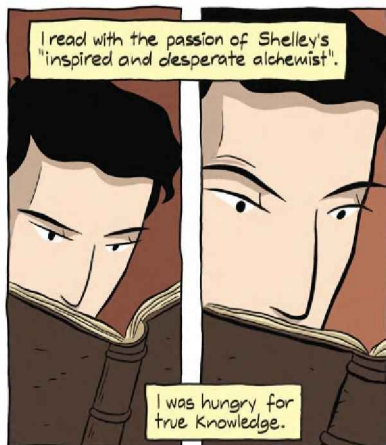




Having earned a Fellowship, I could now pursue a new, alternative itinerary.



I read with the passion of Shelley's "inspired and desperate alchemist".



I was hungry for true knowledge.

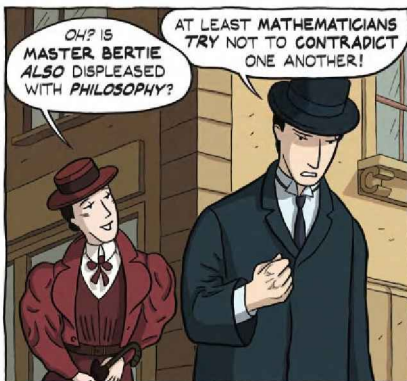
Meanwhile, I persisted in my siege of Alys's heart.



SO, HOW FARES YOUR NEW PARAMOUR?

MM, I'M NOT SO SURE...

OH? IS MASTER BERTIE ALSO DISPLEASED WITH PHILOSOPHY?



AT LEAST MATHEMATICIANS TRY NOT TO CONTRADICT ONE ANOTHER!

NOT SO PHILOSOPHERS! THEY ARE ALL "GREAT" ...AND ALL IN TOTAL DISAGREEMENT!







With my friend Moore, I sought enlightenment at the feet of the latest fashionable Hegelian.

THE TRANSITION FROM THE CATEGORY OF CONTRADICTION SHOWS THAT THE EXCLUSIVE REFLECTION OF THE STABLE POSITION MAKES IT A NEGATIVE, AND THUS THE REFLECTION DEGRADATES ITS PREVIOUS STABLE OF DETERMINATIONS TO THE LEVEL OF BEING ONLY DETERMINATIONS. AND SINCE THE POSITION HAS BEEN MADE POSITION, IT HAS GONE BACK TO UNITY WITH ITSELF.

OH DEAR!

Moore understood me.


THEY CALL THIS TRASH PHILOSOPHY? I WANT TO FIND MY WAY TO REALITY, MAN! I WANT A METHOD TO ACQUIRE CERTAIN KNOWLEDGE!

WELL, HEGEL IS OBVIOUSLY NOT YOUR MAN!

...BUT WHO IS?


IF ONLY PHILOSOPHY HAD A EUCLID!

YOUR BOWLER IS READY, SIR!



SOMEONE TO GIVE IT **STRONG FOUNDATIONS** AND A **LOGICALLY PRECISE LANGUAGE!**

In a hatter's shop, I found at last what I was looking for.




THAT'S EXACTLY WHAT **LEIBNIZ** DID...

...WITH HIS "**CALCULUS RATIOCINATOR!**"



"**RATIOCINATOR?**"



**YES!** A WAY TO MAKE THINKING AS CLEAR AS **GEOMETRY!**



**SO CLEAR** THAT WHEN A **DISAGREEMENT** ARISES, WE JUST HAVE TO SAY...

**ZBRONK**

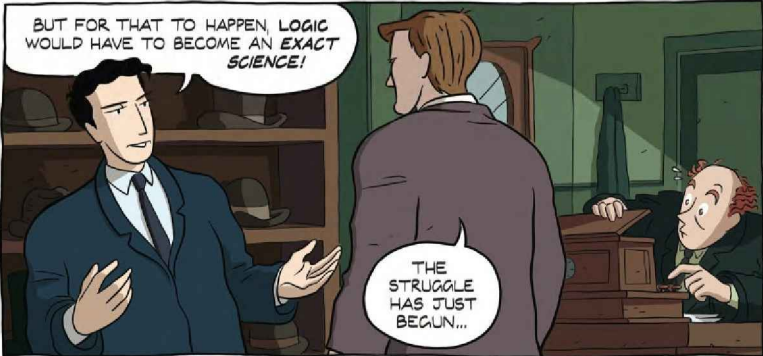
**DRRRRRR**

**CLUNK**



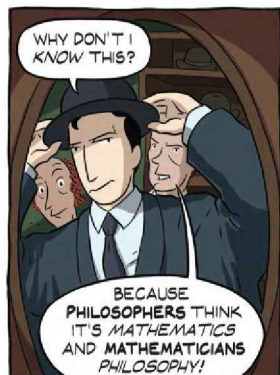
"**CALCULEMUS!**"

"...LET US **CALCULATE.**"



BUT FOR THAT TO HAPPEN, **LOGIC** WOULD HAVE TO BECOME AN **EXACT SCIENCE!**

THE **STRUGGLE** HAS JUST **BEGUN...**





Then and there, Moore introduced us to a new, extraordinary world.

ALL THAT LOGIC REALLY IS, IS USING COMBINATIONS OF THE KNOWN, TO REACH THE UNKNOWN.

THE GREEKS KNEW ALL THAT!

BUT LEIBNIZ USED A FORMAL, SYMBOLIC LANGUAGE TO SAY THINGS LIKE THIS...

$$A \oplus A = A$$

"A TAUTOLOGY ADDED TO ITSELF IS A TAUTOLOGY."

TIME OUT!

"A TAUTOLOGY ADDED TO ITSELF IS A TAUTOLOGY." ISN'T THIS A LITTLE TOO TECHNICAL?

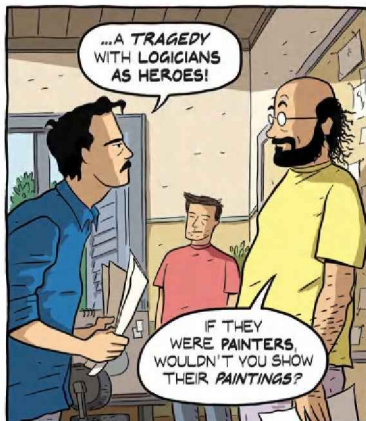
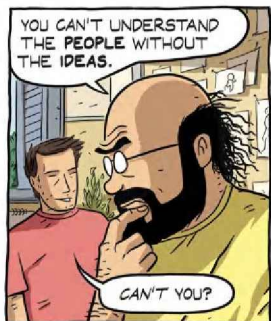
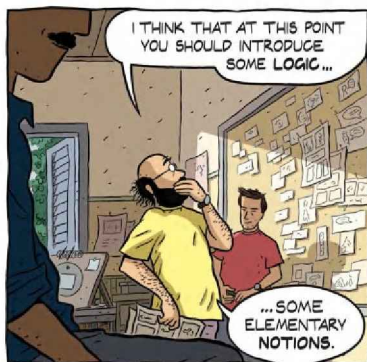
I MEAN, WHAT'S A "TAUTOLOGY"?

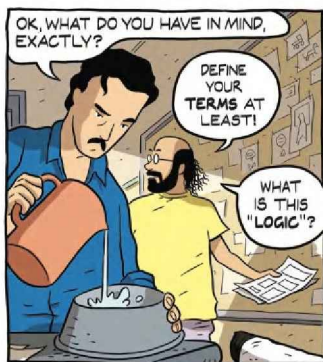
"A TAUTOLOGY IS A STATEMENT WHICH IS NECESSARILY TRUE BY VIRTUE OF ITS LOGICAL FORM, AS IN 'ALL RED ANTS ARE RED.'"

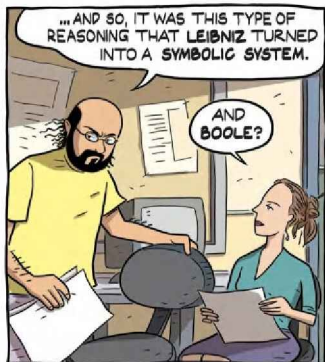
I KNOW WHAT A TAUTOLOGY IS, THANK YOU!

BUT DOES THE AVERAGE READER KNOW THIS?

IS THERE SUCH A BEING?









From the day when I first learned of the dream for a purely logical calculus, I was hooked. This new fascination completely took over my life...

...Well, almost completely!

BERTIE?

Even in an ideal City of Reason the irrationality of Eros will creep in.



IS THE MAN YOU ARE READING SO FASCINATING?

FASCINATING ENOUGH...



...TO BE THE HERO OF THE MAN YOU ARE READING!



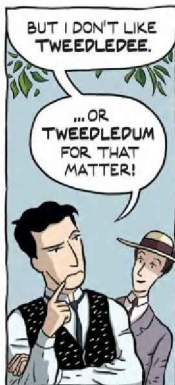
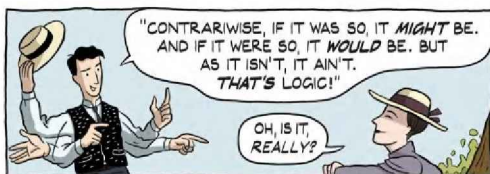
OH?

INDEED! "LEWIS CARROLL", a.k.a. MR. DODGSON, IS AN EXPERT IN BOOLE'S IDEAS!



THE SAID BOOLE BEING THE MAN WHO HAS MADE LOGIC AS CLEAR AS ALGEBRA!

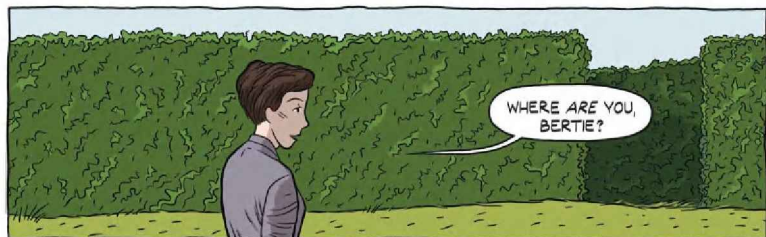
YOU DON'T SAY!

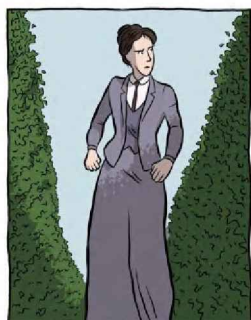


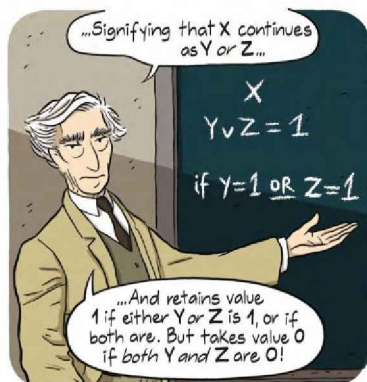
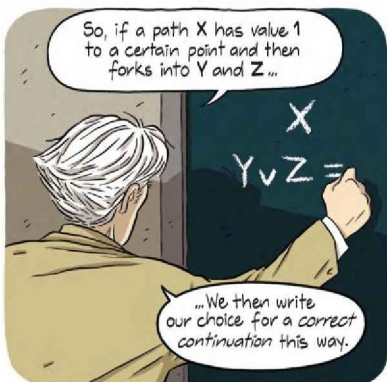
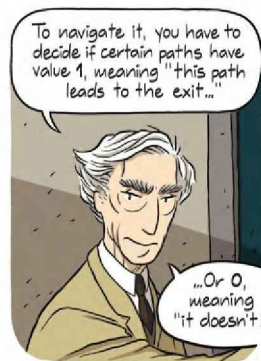


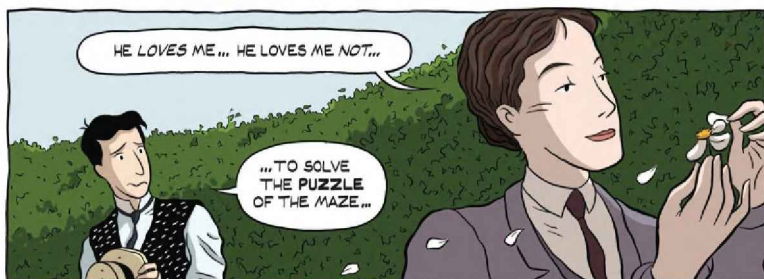
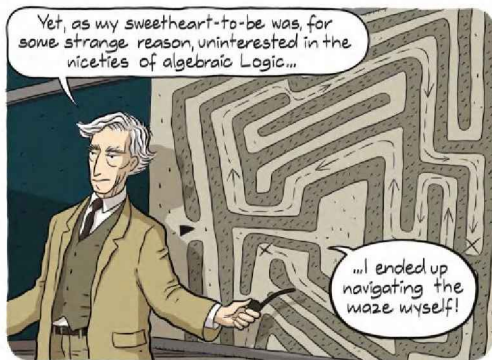


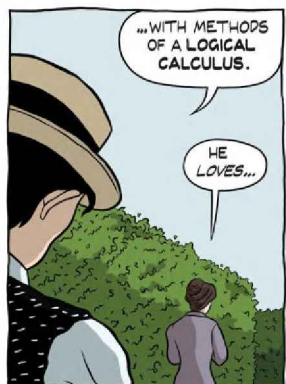












...My impromptu attempt at a lesson ended with me as the student.

Mmmmmmmmm...



And though unprepared, I did rather well!

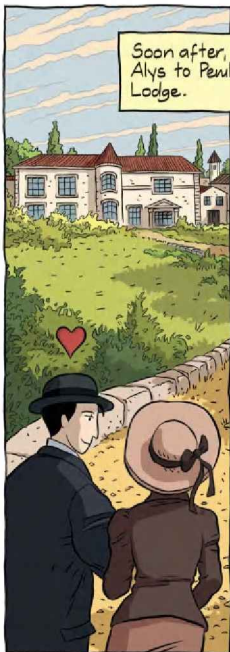
Being no expert in feminine psychology, I decided their meeting was a huge success.

OH, BERTIE, I THINK SHE HATED ME!

DON'T BE SILLY, OLD GIRL.



Soon after, I took Alys to Penubroke Lodge.



GRANDMAMMA, I PRESENT MISS ALYS SMITH!



SHE ADORES YOU!

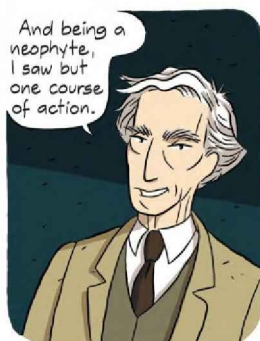
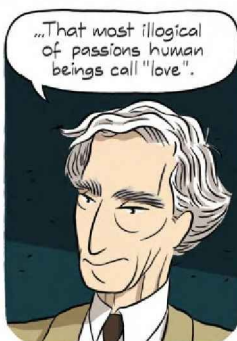


DO YOU THINK SO?

AND SO DO I!!!



BERTIE!



But though my love life was at last waking headway, my career as a thinker was stalling.

DON'T LOOK SO GLUM, OLD CHAP, YOU NOW HAVE AN AIM: TO LEARN LOGIC!

I'VE BEEN "LEARNING LOGIC" FOR THE PAST YEAR...

...BUT IT'S HARDLY ENOUGH TO COVER MY NEEDS!

LEARN MORE THEN!

THERE ISN'T ANY...

To understand my predicament, remember that my profound, underlying aim had never changed: to acquire certain knowledge about the world...

...Knowledge which could only come from Science.

But Science depended on Mathematics, which was a total mess, plagued by unproven assumptions and circular definitions. To repair it, a powerful Logic was needed...

...But there wasn't one! And so we came to an impasse.

By now, I'd come to realize that Mathematics resembled the Cosmos of Indian Myth: its apparent solidity really depended on the reptilian whims of its carrier. Mathematics rested...

...On shaky foundations!





The sorry state of the "Queen of the Sciences" was made even worse by the successes of Physics.

THE WORK OF THOMSON AND RUTHERFORD IS TRULY REVOLUTIONARY!

WE ARE APPROACHING THE DEMOCRITTEAN VISION, THE DISCOVERY OF THE ATOMS OF MATTER!



...BUT OUR POOR MATHEMATICS LAGS LAMELY BEHIND!

AND WHAT'S WORSE IS THAT MATHEMATICIANS DON'T FACE UP TO THE PROBLEM!

ONLY THEN CAN WE BEGIN TO SET THE HOUSE OF MATHEMATICS IN ORDER!!!



With Moore, I dreamed of great discoveries.

WE MUST MAKE THEM REALIZE THE TERRIBLE MESS!



AWFULLY SORRY TO BARGE IN, CHAPS, BUT YOU ARE TALKING BALDERDASH! MATHEMATICS IS IN PERFECT ORDER!



The situation shocked me: most mathematicians were painfully unaware of the flimsiness of the foundations.

THEY ARE FOOLS, MOORE!

DON'T LOSE HEART, OLD MAN!



Yet my despair did not last. For finally I met a man refreshingly rigorous in his approach.

TO ACHIEVE ANY KIND OF CERTAINTY IN MATHEMATICS, WE MUST RE-EXAMINE ITS BASIC ASSUMPTIONS, WE MUST BEGIN AT THE BEGINNING.

PROPOSITION XV. If  $x \neq y$ , then  $y \neq x$ .  
PROPOSITION XVI. If  $z \neq xy$ , then  $z \neq x$ ,  $z \neq y$ ,  $z \neq x + y$ .  
PROPOSITION XVII. If  $z \neq xy$ , then  $\bar{x}\bar{y} \neq \bar{z}$ ,  $\bar{x} + \bar{y} \neq \bar{z}$ .  
PROPOSITION XVIII. If  $z \neq x + y$ , then  $z \neq x$ ,  $z \neq y$ ,  $z \neq xy$ .  
PROPOSITION XIX. If  $z \neq x + y$ , then  $\bar{x}\bar{y} \neq \bar{z}$ ,  $\bar{x} + \bar{y} \neq \bar{z}$ .  
PROPOSITION XX. If  $xz \neq y$ , and  $x \neq y + z$ , then  $x \neq y$ .

HEAR HEAR!

In Alfred Whitehead, I found a strong and kindred spirit.

...A mentor.

OH THERE ARE PEOPLE WHO CAN SEE THE SITUATION CLEARLY, RUSSELL. BUT ALL OF THEM ARE, ALAS, ON THE CONTINENT.

IF WE UNITE THE HEALTHY PARTS OF MATHEMATICS AND THE CONCEPTUAL SOPHISTICATION OF THE NEW LOGIC, WE CAN LAUNCH A POWERFUL ATTACK.

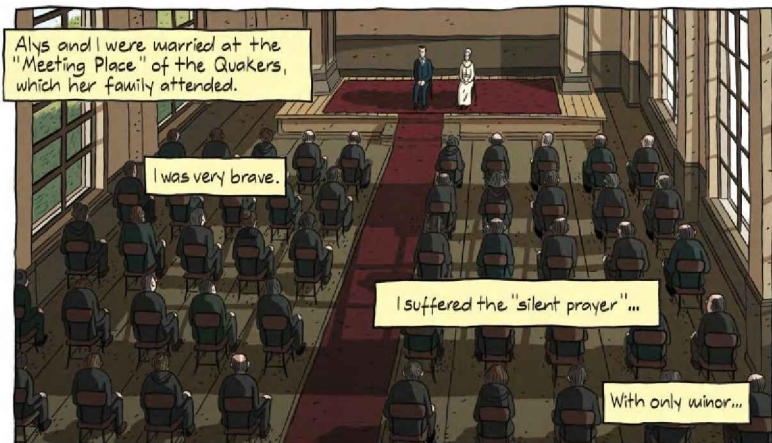
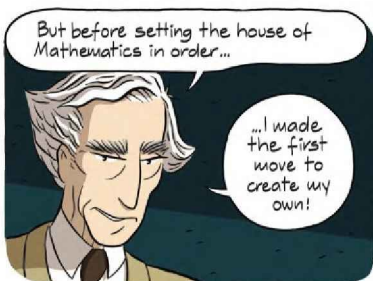
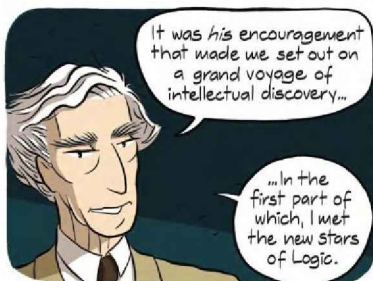
"NEW"? LOGIC HAS NOT ADVANCED ONE BIT AFTER BOOLE!

SURE, IT HAS COME A CERTAIN WAY SINCE OLD ARISTOTLE ...

BUT IS IT STRONG ENOUGH YET TO DEAL WITH MATHEMATICS?

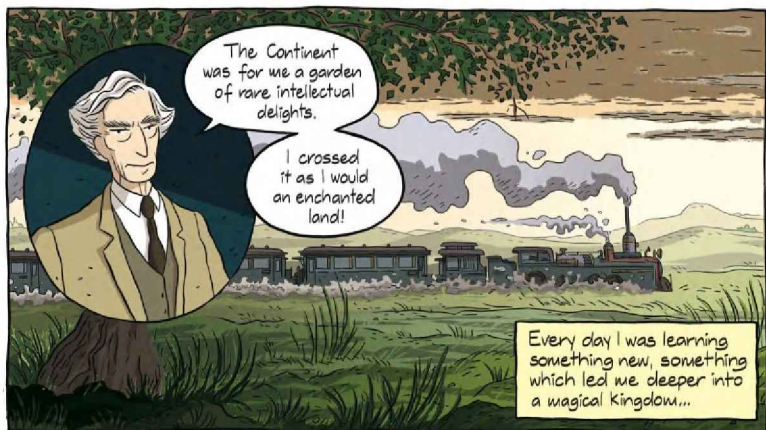
THERE IS AN OLD GERMAN SAYING: "IF YOU WANT TO LEARN SOMETHING, GO ON A JOURNEY!"

It was Whitehead, more than anyone else, who helped us see beyond the provincialism of the English mathematical establishment.



### 3. WANDERJAHRE





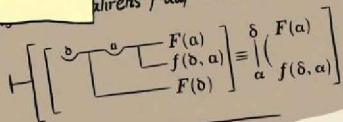
However, neither he nor his work was well known. But that didn't make me doubt its potential importance. Truth is...

**BRIFFSSCHRIFT**

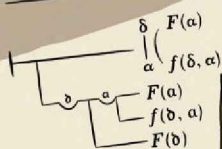
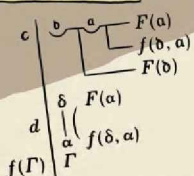
die sich in der f-Reihe vererbt, so hat jedes  
 Element f auf x die Eigenschaft F.

Ergebnis einer Anwendung

69



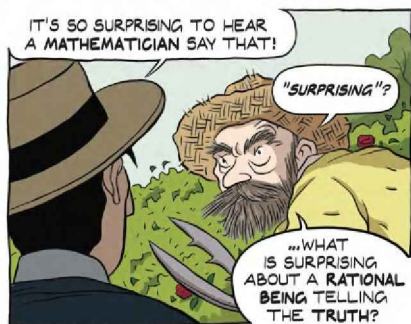
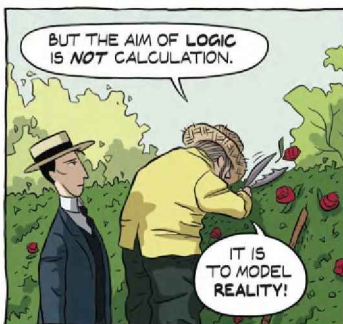
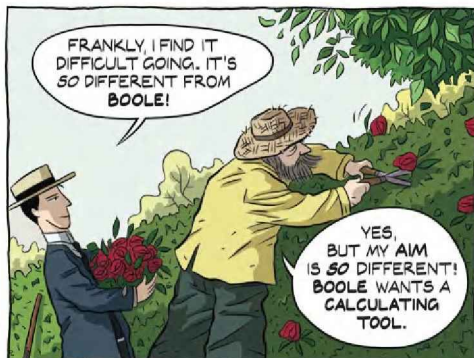
...The "concept script" did not look very inviting!

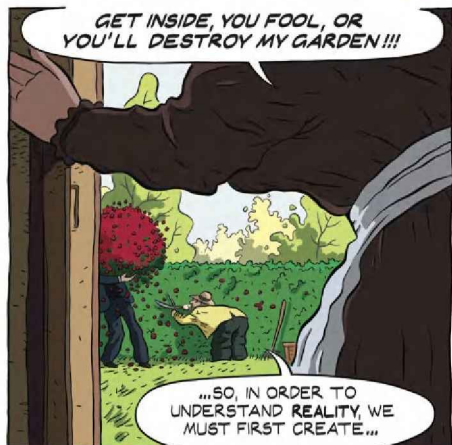


...Yet, once the abstruse surface was penetrated, a lot of sense could be found underneath.

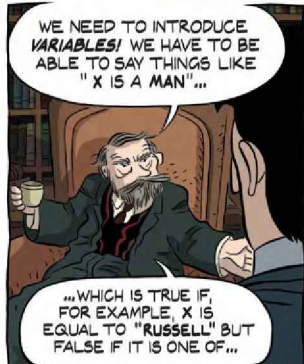
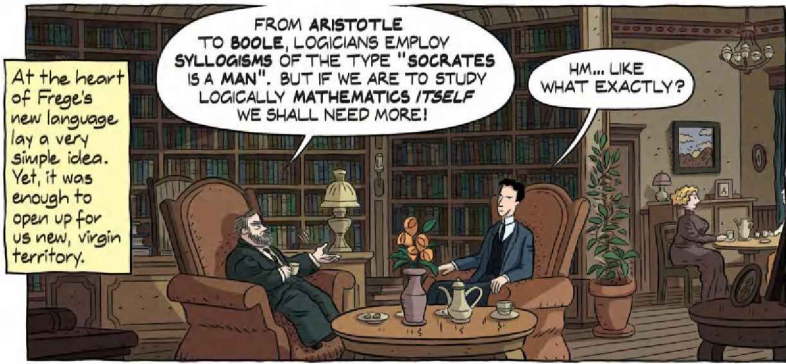
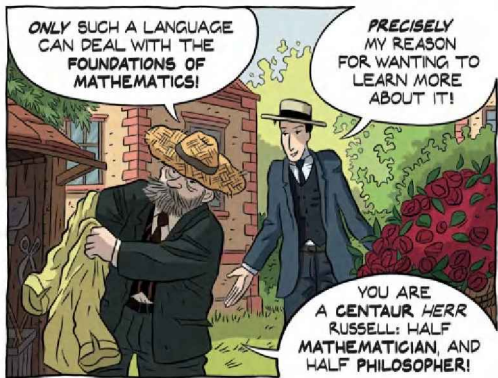
Wenn aus dem Satze, dass b die Eigenschaft F hat, was auch b sein mag, werden kann, dass jedes Ergebnis einer Anwendung des Verfahrens f in der f-Reihe vererbt sich die Eigenschaft F in der f-Reihe.













From Jena,  
we moved  
to our next  
destination.

OH, BERTIE...

WILL YOU BE LIKE  
THAT SOME DAY?

HM?

YOU MEAN LIKE FREGE?  
I CERTAINLY *HOPE* SO!

YOU DO?

YOU SEE HIM AS AN OLD  
ECCENTRIC, BUT HE IS  
A *GREAT MAN*!

I WOULDN'T  
WANT TO BE THE  
"GREAT MAN"'S  
WIFE!

BUT WHAT IF HIS  
SMALL QUIRKS ARE  
THE OTHER SIDE OF  
HIS *GENIUS*?

WHAT IF HIS RIGOUR IN BIG  
THINGS IS THE EXTENSION  
OF HIS PASSION FOR  
EXACTITUDE...

...IN  
LITTLE  
ONES?

ANYWAY, LET'S WAIT A BIT BEFORE  
WE PASS JUDGMENT ON THE  
HABITS OF GIANTS.

THIS TALK  
OF "GIANTS"...  
LIKE FAIRYTALES ...









Let me now make a general remark regarding this rather sensitive subject...



...Mathematics.

All of you have some experience from school. Those who dislike it, see it as sheer drudgery. Those who don't, see it perhaps as a game...



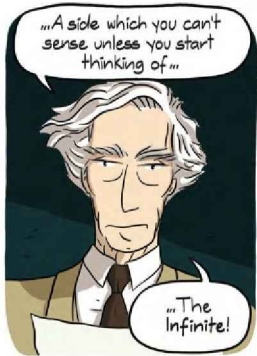
And there is an element of that in it.



But there is also another side to Mathematics...

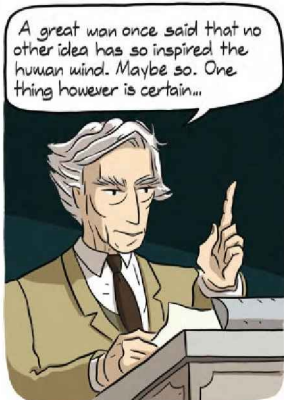


...A side which you can't sense unless you start thinking of...

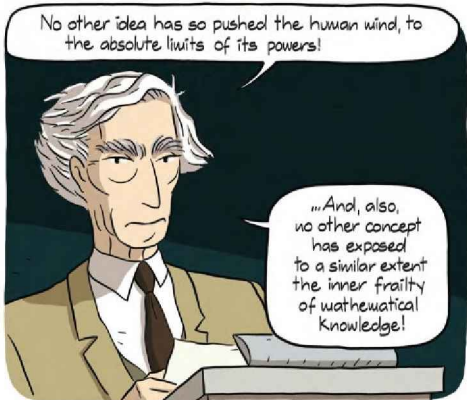


...The Infinite!

A great man once said that no other idea has so inspired the human mind. Maybe so. One thing however is certain...



No other idea has so pushed the human mind, to the absolute limits of its powers!



...And, also, no other concept has exposed to a similar extent the inner frailty of mathematical knowledge!



It was for this reason, really, that Gauss warned against frontal attacks on infinity.

Yet, his stern warnings did not enter my mind on that morning...

...As I set out to meet Georg Cantor, the Magus of the Infinite!



Here I was, a Briton who had set out in search of German wisdom.

On the way, I ran into a fellow who had made the trip in the opposite direction.



GEORG  
FRIEDRICH  
HÄNDEL



HAAALLELUJAH!  
HALLELUJAH!  
HALLELUJAH!

I headed for my destination in high spirits.

Mind you, Halle's University had recently incorporated Wittenberg's and thus could lay claim to being the *Alma Mater* of both Hawlet and Doctor Faust. And so it had...

...Perfect credentials for trouble!





I headed for the new location indicated, certain that Cantor had left the University...



...For a higher academic position.



So, as I reached a group of dark buildings...

...I surmised they belonged to an institution for the New Mathematics.



THIS WAY FOR HERR PROFESSOR CANTOR?

JA, JA, FOR ALL THE "PROFESSORS"!



If this was so, the decrepit state of the interior...

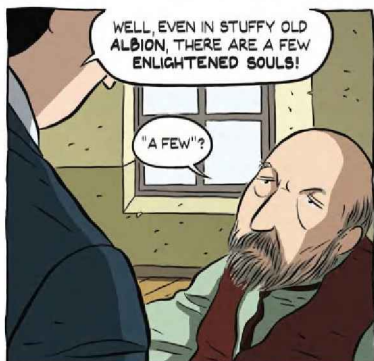
...Spoke badly of the state of the New Mathematics!

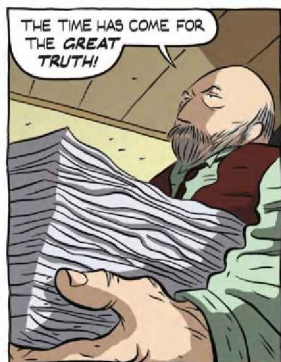
Try and imagine a young painter being received by Michelangelo.

A composer meeting Beethoven.



That is how I felt, as I knocked on Cantor's door.







AH... ER...

Madness had always terrified me. But to see it take over a great mind, was devastating.



I... I REALLY MUST GO...

THE CONSPIRACY IS EXPOSED!



YOU MUST GO TO THE QUEEN!!! SHE MUST...?



...PROTEST MY IMPRISONMENT!

NOW, BE A GOOD BOY PROFESSOR!



I AM HELD CAPTIVE AGAINST MY WILL!



I SPEAK THE WORDS OF THE PROPHET!!!



"...I WILL BLOW IN THE FIRE OF MY WRATH!!"

OOHHH

I fled the asylum with a dark leitmotif from my childhood roaring fortissimo...



I was too confused to attempt any explanation.







**TRAITOR!**

WHA... WHAT HAVE I DONE?!

**YOU MESSED WITH INFINITY!**

BUT...

BUT... THAT'S...  
NOT HÄNDEL, THAT'S...  
THAT'S...

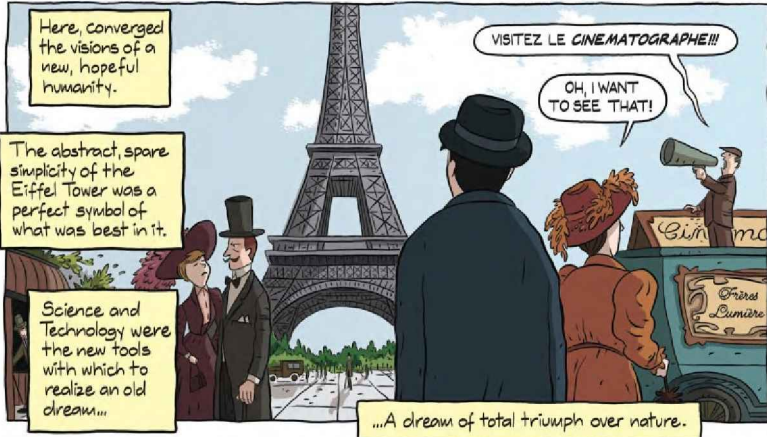
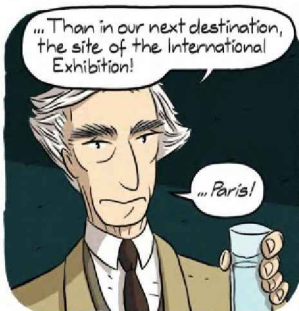
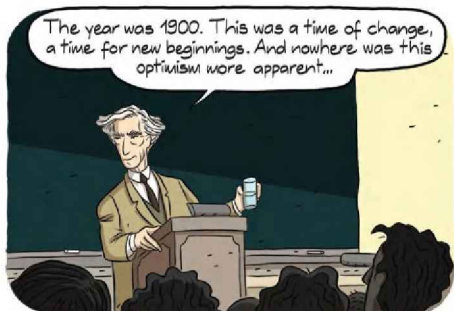
**Gauuuss!!!**

**YOU WRECKED  
THE FOUNDATIONS!!!**



OOOO





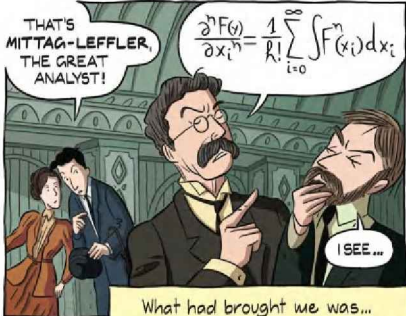
The visitors to the Exhibition were like children, excited with the toys of a brave new world...

... Toys which played in unexpected ways!



But we had not come to Paris for the Exhibition.

Or, rather, I personally hadn't!



...The International Congress, the world's greatest rendez-vous of Mathematicians.

Everybody who was anybody in Mathematics was there!



...Not all encounters in Paris were wathematical!



But I was too excited by new ideas to pay serious attention to new emotions.



And there was no shortage of new ideas: new theories, new techniques, new methods. There was even...



...A host of new fields!

Yet my own interest was focused in a single direction...

...The new logical language that would give Mathematics solid foundations.

It was clear that a central role in this would be played by the Theory of Sets.

Géométrie Différentielle  
Salle 1 3<sup>ème</sup> Etage  
Géométrie Elliptique  
Salle 13 Rez-de-chaussée  
Géométrie Hyperbolique  
Salle 5c 1<sup>er</sup> Etage  
Géométrie Projective  
Salle 3 1<sup>er</sup> Etage



...On whose value the Congress's two greatest stars vehemently disagreed!

## L'AFFAIRE CANTOR

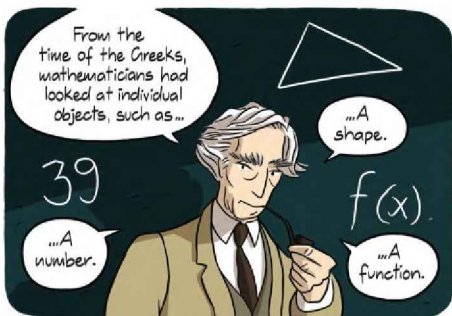
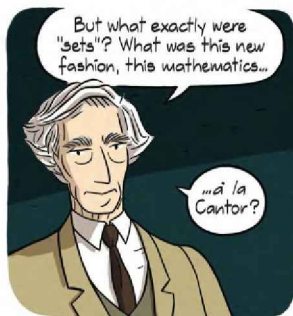
J'accuse Monsieur Cantor!  
Set Theory is a disease from which  
Mathematics must be cured!

No one shall expel us from the Paradise  
Herr Cantor has created for us!

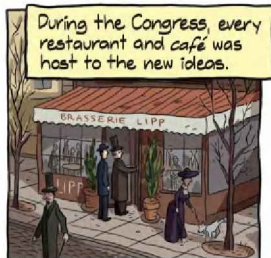


Henri Poincaré, the great French genius, a strong believer in the importance of human intuition.

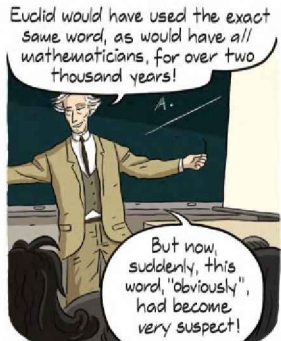
David Hilbert, no less great, the German apostle of the rigorous exactness of logical proof.











The advent of the new, non-Euclidean geometries, had subverted the notion of axioms as "obvious truths". In fact, it had supplanted the very notion of "obviousness"!

WHO WOULD NOT LIKE TO LIFT THE VEIL AND SEE THE FUTURE, WITH THE NEW FACTS AND METHODS IT WILL DISCLOSE?

This spirit was nowhere better expressed than in Hilbert's much-expected talk on "the Problems of Mathematics".

His stated aim was to give a bird's-eye-view of the future, by way of twenty-three major unsolved problems.

INTUITION SHALL NO MORE HAVE PLACE IN OUR PROOFS!

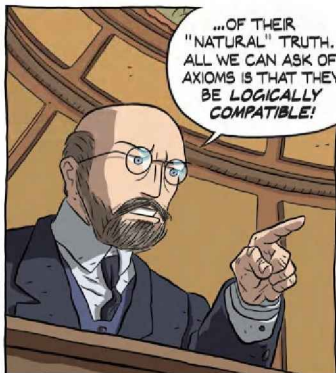
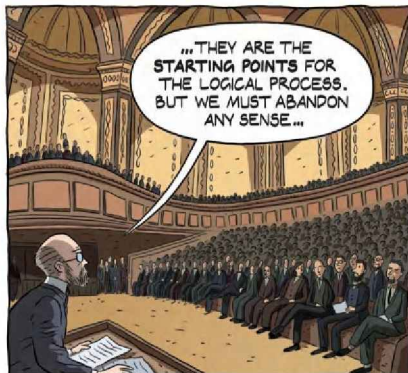
THE NEW MATHEMATICS WILL NOT ADMIT ANYTHING AS "INTUITIVELY OBVIOUS"! NO TRUTH EXISTS FOR US OUTSIDE THE CRUCIBLE OF RIGOROUS PROOF!

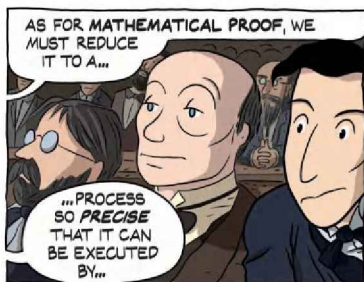
Yet, his speech was above all a plea for a new mentality, which annulled a host of existing preconceptions.

AS FOR THE AXIOMS OF A THEORY...

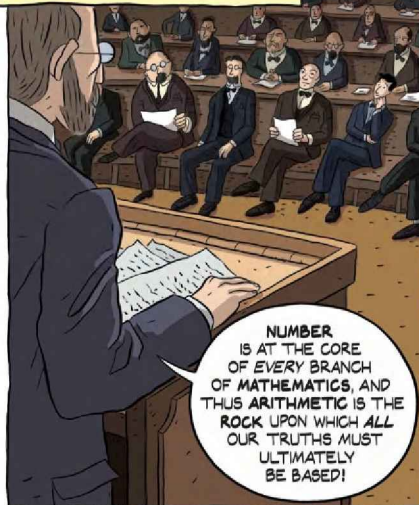
...THEY ARE THE STARTING POINTS FOR THE LOGICAL PROCESS. BUT WE MUST ABANDON ANY SENSE...

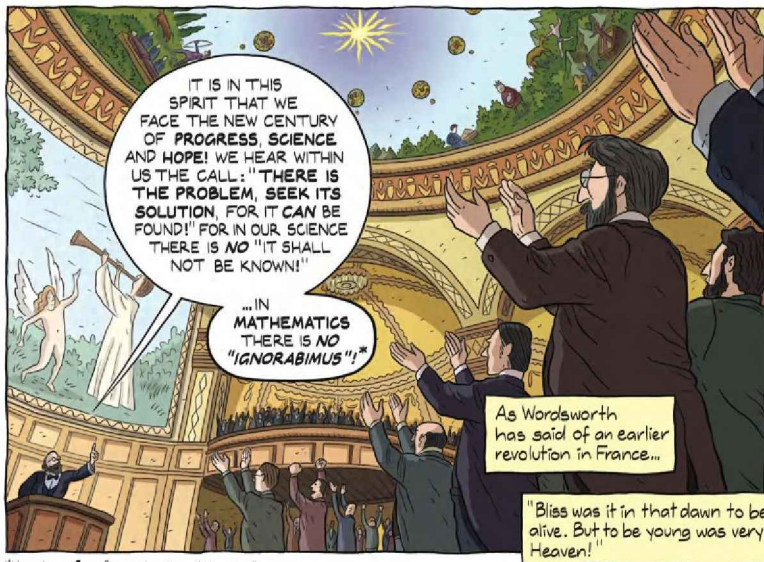
...OF THEIR "NATURAL" TRUTH. ALL WE CAN ASK OF AXIOMS IS THAT THEY BE LOGICALLY COMPATIBLE!





Some of "Hilbert's Problems" of 1900 keep mathematicians busy even today. But one of them became the target of my own dreams.





\* Latin for "we shall not know."

I crossed the Channel with my heart firmly set on the course it was to follow henceforth. But really, I had come full circle, to my first intellectual frustrations.



At last, I had to face my disillusion with Euclid's "obvious" axioms head on.



PENNY FOR YOUR THOUGHTS?...



OH... I DON'T THINK THEY ARE WORTH THAT MUCH... YET!

TRY ME...



AHEM... WELL, LET ME SEE... FREGE AND THE ITALIAN PEANO...

...CREATED A THEORY ABOUT NUMBERS... ER... LET ME SEE HOW TO PUT IT...



IS THERE A BLACKBOARD ON BOARD?

HA HA.



HEY!





## 4. PARADOXES





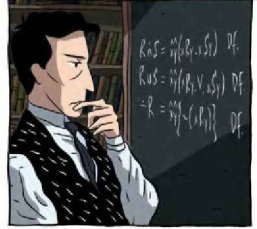
After my return from Paris, I set out with fiery, though rather misjudged, optimism, to write the book that would solve all foundational problems — and then some!

This was "The Principles of Mathematics", my first go at becoming a new and greater Euclid!

I built on the ground created by Frege in volume one of his "Foundations of Arithmetic".

I used an elegant notation invented by Peano.

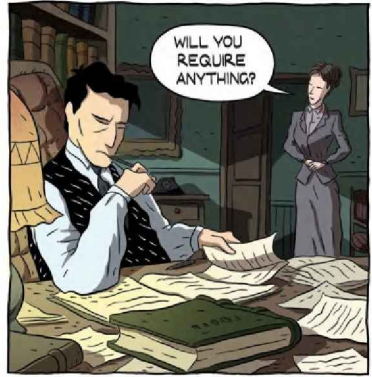
I was convinced I was on the right track.



Hard work was all I needed to reach my goal.



BERTIE?



WILL YOU REQUIRE ANYTHING?



FREEDOM FROM FURTHER INTERRUPTIONS.

GOODNIGHT.

The treasures of Logic came at a price.

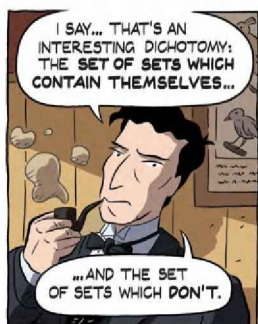




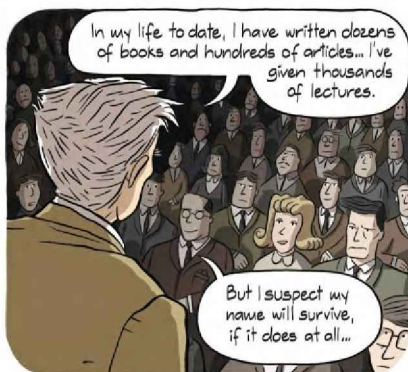


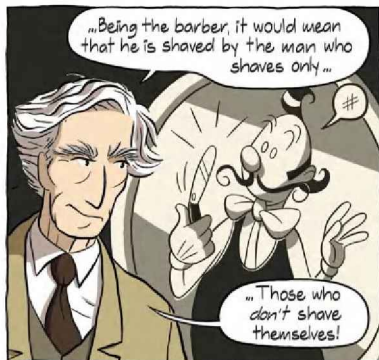
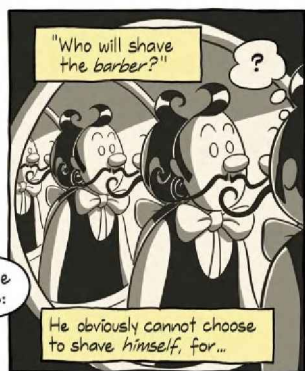


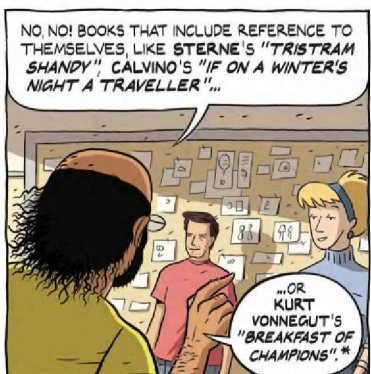
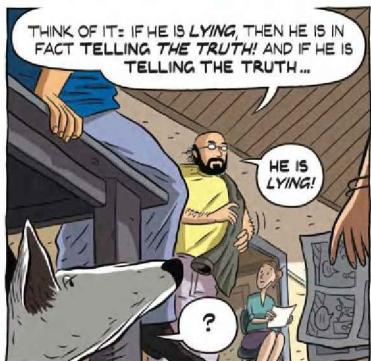
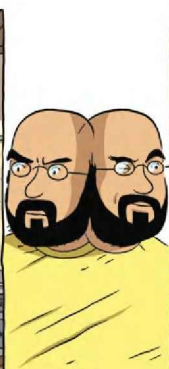






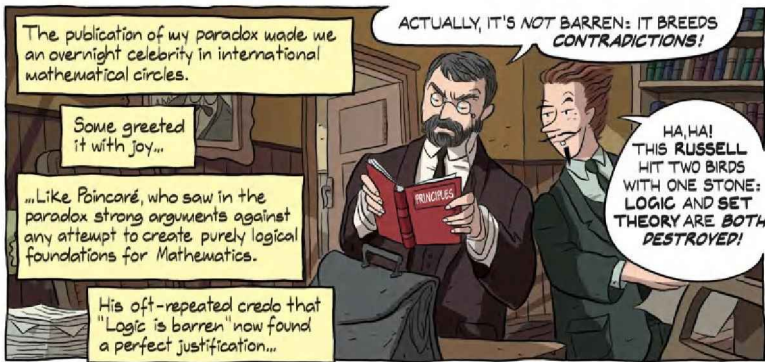
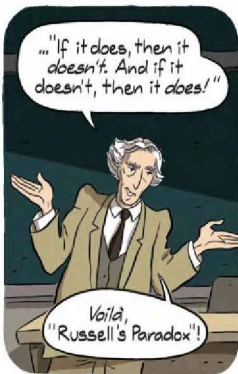






\* Of course LOGICOMIX is also self-referential.









Gottlob Frege  
in Jena.

He read my paradox  
on the very day when  
he was to give the  
go-ahead to print  
volume two...

...Of his "Foundations of Arithmetic".



In an instant,  
he realized the  
import of my  
discovery.



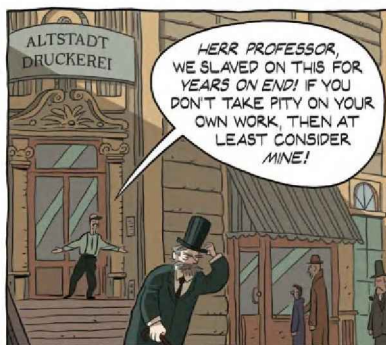
Frege, too, had built  
his edifice on the  
ground of Bolzano's  
simple idea of set.

And now he had seen that this ground was rotten — it had given way.



By implanting sets into Logic, he  
had injected a lethal canker into  
its body. So: the "Foundations of  
Arithmetic" were... unfounded.

DON'T BE  
LATE FOR  
DINNER,  
GOTTLÖB!



In the end, he did publish volume two of the "Foundations of Arithmetic". But with an addendum.

### ADDENDUM

Hardly anything more unfortunate can befall a scientific writer, than to have one of the foundations of his edifice shaken after the work is finished. I was placed in this position by a letter of Mr. Bertrand Russell, just when the printing of this volume was nearing its completion.

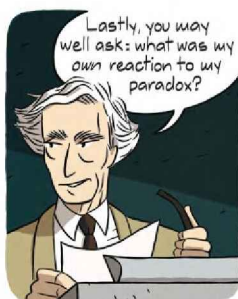
Of all the acts of intellectual honesty I have witnessed in my life, none compares with Gottlob Frege's reaction to my paradox.

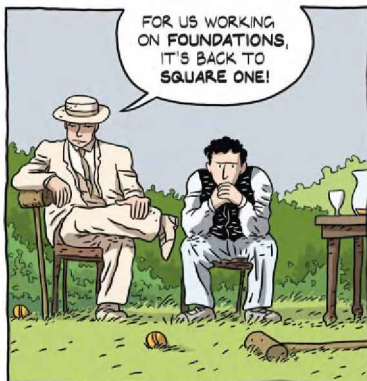
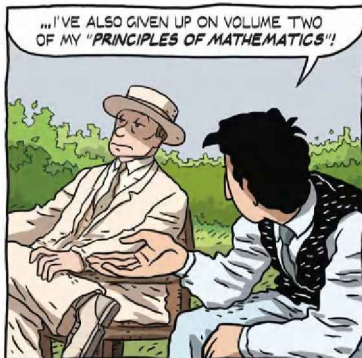
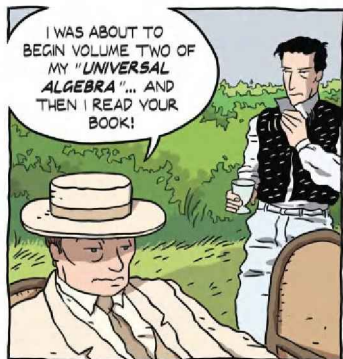
There cannot be greater intellectual courage than this...

The collapse of one of my laws, to which Mr. Russell's paradox leads, seems to undermine not only the foundations of my Arithmetic but the only possible foundations of Arithmetic as such.

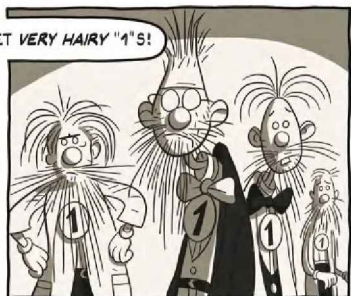
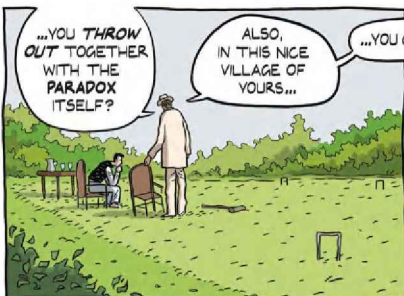
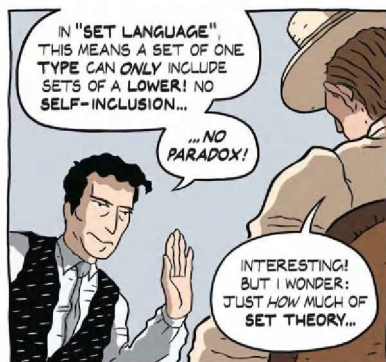
... To put the Truth above all else.

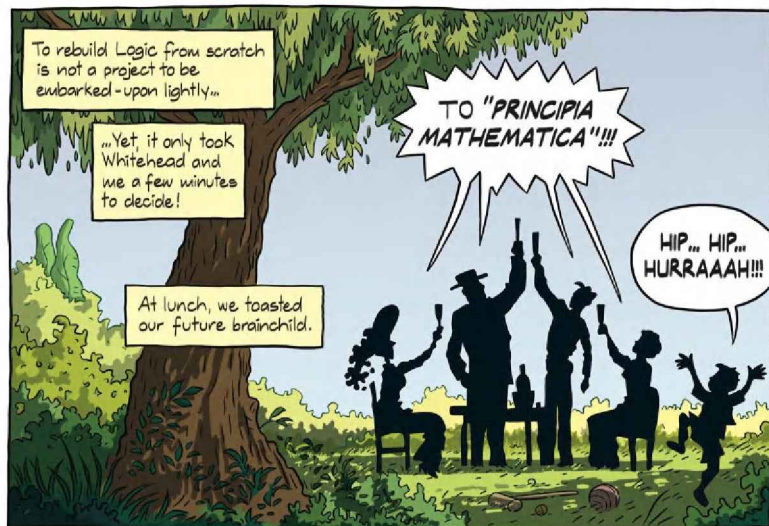












Spring 1905 came and went, and our work's completion was nowhere in sight...

HEY, WHITEHEAD...  
...OPEN UP!

MY DEAR FELLOW...  
ARE YOU FEELING ALRIGHT?

...AFTER A NIGHT OF COMPLETELY RE-WRITING CHAPTER THREE!

AS WELL AS I CAN...

!

THE THEORY OF TYPES IS ROT!

BUT OUR WHOLE ARGUMENT IS BUILT ON TYPES!

AND? IF THE PREMISE IS WEAK, SO IS ALL THE REST!

...SO, LET'S DROP IT!

OH, RUSSELL...

WE WORKED FOR TWO YEARS ON "SIMPLE" TYPES...

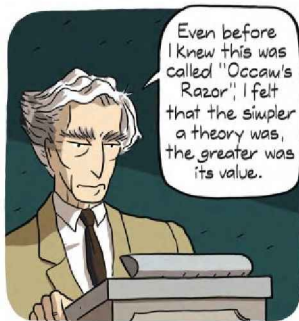
...ONLY TO DUMP THEM FOR THE "RAMIFIED".

THOSE GAVE US SOME HOPE, AT FIRST, BUT...

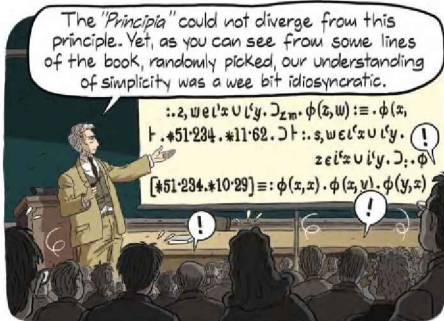
BERTIE...  
WHAT A PLEASANT SURPRISE!







Even before I knew this was called "Occam's Razor" I felt that the simpler a theory was, the greater was its value.



The "Principia" could not diverge from this principle. Yet, as you can see from some lines of the book, randomly picked, our understanding of simplicity was a wee bit idiosyncratic.

$z, w \in U \setminus y, \exists z, \phi(z, w) \equiv \phi(z, t, *51*234, *11*62, \exists t :: s, w \in U \setminus y, z \in U \setminus y, \exists \cdot \phi$   
 $(*51*234, *10*29) \equiv \phi(x, x), \phi(x, y), \phi(y, z)$



Whitehead and I spent that summer re-examining our premisses.

...SO IF WE TAKE PREDICATE "P" TO STAND FOR...



And by the time autumn came, we had changed course again.

AND SO REDUCES TO...

Our new trick was as old as Euclid: a new set of axioms!

Once again, we rebuilt from the bottom.



To gain more time for work, my wife and I moved in with the Whiteheads.

We found the change most pleasing - well, at least half of us did!

Yet, living under the same roof was not a cure for our problems...



The endless hours spent at our desks resulted in a stronger language.

2006

CRAAAACK

THIS CAN'T GO ON, RUSSELL!

But the project's central problem was always there.

2001  
1998

The deeper we got into our Quest...

... The more I doubted its premises.

ISN'T IT OBVIOUS TO YOU? WITHOUT **SECURE FOUNDATIONS**, WE CANNOT BUILD OUR SYSTEM!

I KNOW, BUT I CAN'T STOP MYSELF FROM ASKING—

"ASKING"?

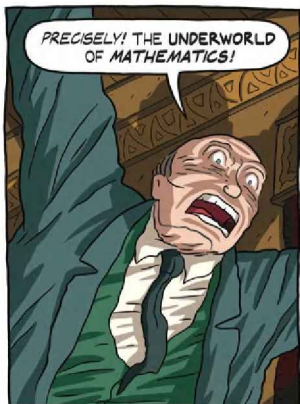
OUR WORK IS NOT ABOUT ASKING, MAN...

... IT'S ABOUT ANSWERING!

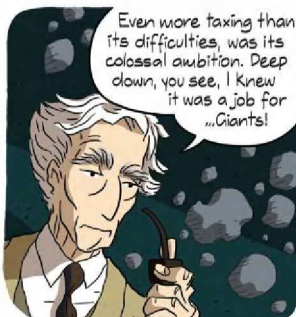
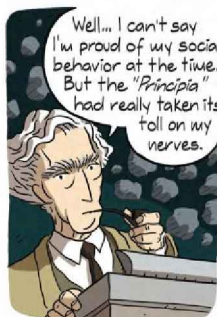
OH MY GOD...

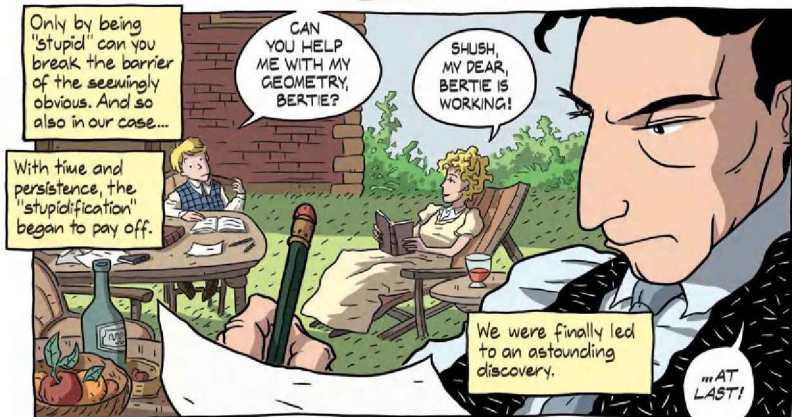
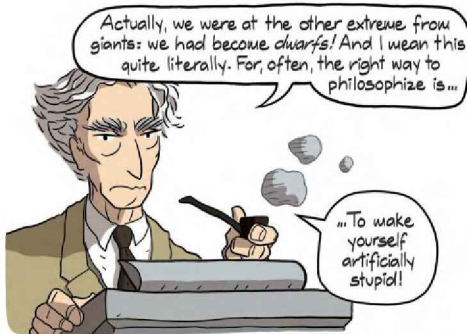
WHEN WE **STARTED**, ERIC COULD BARELY COUNT TO THREE... AND NOW HE DOES **THREE-DIGIT MULTIPLICATION!**

AND THIS STORM WON'T STOP!





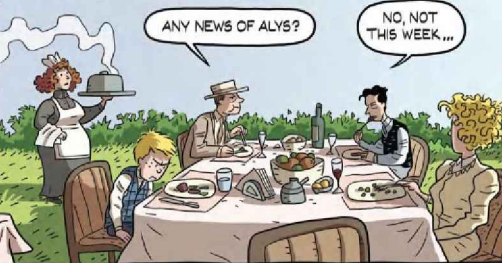




To achieve this monumental task, took us a mere... 362 pages! Think of that: 362 pages to prove what every child knows.



During the last phase of our work, my wife spent most of her time in rest homes.



I often think back at my stay with the Whiteheads...

Halcyon days.



It took us ten years to complete the first three volumes of our grand edifice.

Though, actually, I didn't know at that time that "first" would also mean "last".



I'M  
TIRED  
MAN.


In fact, I didn't even know, back then, that we had completed anything.

I'M  
TOTALLY  
WRECKED.

COME...



THERE'S  
SOMETHING  
I WANT YOU  
TO SEE.

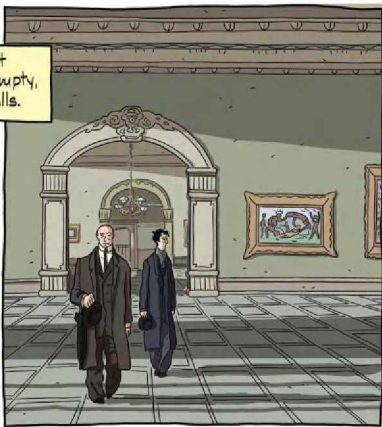


NOW SHOWING  
THE PRERAPHAELITE  
BROTHERHOOD AND THEIR  
INFLUENCE

Whitehead had found a perfect symbol for his argument.



He led us to it  
through the empty,  
resounding halls.



THERE...



THE DANAIDES?

CONDEMNED BY THE GODS,  
ENDLESSLY TO POUR WATER,  
TO FILL A LEAKING JAR!





Dear friends, I know well that, despite anything Socrates may have believed...



...Lay people often feel wiles away from a philosopher's worries.



So, I want to ask:

Can you at all understand the state I was in, back then? Does my desperation make any sense to you? ...What say you, Madam?



Well, I'll admit that it's not too clear, Professor Russell!

Alright.. Let's go over the stages of the journey so far: "1" is the need that started it...



1. MATHEMATICS MUST BE BASED ON LOGIC!
2. FREGE CREATES THE RIGHT LOGIC (BASED ON SETS)
3. I FIND PARADOX i.e. "LOGIC IS FAULTY!"
4. WHITEHEAD & I MUST FIX IT ("PRINCIPIA")

... "2" made the quest possible... "3", which marks my own entry, is the major crisis...

...And "4", the struggle to overcome it!

So, what Whitehead and I were really doing, in building a paradox-free Logic that could support Mathematics, was...



...Fixing the hole I had exposed in Frege's ideas!

And we were successful in our task, in all ways but one: no matter how deep we went, our too-too-solid system was being built on sand. Or worse...



I've said that the Foundations of Mathematics were like a mythical turtle supporting the Cosmos. Yet, all we did when we tried to create solid ground for the beast to stand on... was...



...A tower of "turtles", all the way down!

To me, a philosopher, the irony of the situation of a "foundational system without foundations" was hard to bear...



...Too hard!

Yet, despite my initial reticence to publish, I eventually agreed: Maybe a book would help us find new associates in our efforts!



Also, of course, I suffered from a bad case of intellectual cabin fever...



...And publication offered a way out of my prison!



"I WEEP FOR YOU, THE WALRUS SAID..."

DON'T BE A SPOILSPORT, BERTIE. THIS IS EXCITING!

I walked with Whitehead to the publishers.



THE "PRINCIPIA" SHOULD HAVE LED US INTO PARADISE...

...BUT WITHOUT A SOLID FOOTING IT'S MERELY A COMPETENT SURVEY OF HELL!



TUSH, RUSSELL, YOU'RE JUST GETTING COLD FEET.

ALEA IACTA EST!



I didn't go in.





Suddenly, I'd realized that I was following the wrong track.



EVELYYYYNN!!!



WHAT ON EARTH???

I... I... SEE IT... GASP...



IT'S... IT'S NOT... NOT THE BOOK!

WHAT ARE YOU TALKING ABOUT?

My skills as a communicator had not yet reached their present level.



YOU'RE SUCH A MESS!

...AND I KNOW... NOW, I KNOW WHY...

...IT'S ALL WRONG!



WHAT IS?



WELL...

IT'S...

IT'S...









Ten years of daydreams of the triumph of our opus magnum had come to this.

THEY COULDN'T FIND A SINGLE READER TO EVALUATE THE MANUSCRIPT, SO THEY FIGURED: "IF NO ONE WILL ACCEPT TO READ THE 'PRINCIPIA' AND BE PAID FOR IT..."

"...THEN, OBVIOUSLY, NO ONE WILL PAY TO READ IT, EITHER!"

The publishers' thinking was pretty sound.

But, convinced that the "Principia" should enter the community of ideas, we decided to accept the ignominy of paying to see our work in print.

STAY AWAY FROM THE MUD, KURTI!

And one final thing on this sad day, and its parallel emotional misadventures...

In the thirty years since it was published, I've only met one person who I'm convinced has read the two thousand or so pages of forbidding, symbol-packed text, cover to cover.

But he was only a child back in 1910.

With hindsight, I say that I was wrong in my self-analysis: my problem was the book!

# ENTRACTE







WELL, IF THE "FOUNDATIONAL QUEST" IS, AS YOU IMPLY, A "SPIRITUAL TRAGEDY" OF SOME KIND, THEN IT SHOULD HAVE A MORAL!

ARISTOTLE SAYS THE ACTION OF A TRAGEDY IS "COMPLETE IN ITSELF"...

...i.e. FOR A MORAL YOU NEED AN ENDING.

BUT OUR STORY IS NOT FINISHED YET!


OK, SO, WHERE IS IT HEADING?

IT'S NOT JUST THE FINAL DESTINATION THAT'S IMPORTANT... IT'S THE ROAD!

THE MEANING IS IN EVERY TURN THE HEROES MAKE, EVERY STOP, EVERY CUL-DE-SAC...

IN A SENSE THE FOUNDATIONAL QUEST IS AN INCOMPLETE ODYSSEY!

AH, YES! WITH LOGICAL CERTAINTY IN THE ROLE OF ITHACA!



ALSO, I MUST SAY I'M NOT REALLY COMFORTABLE WITH THE "LOGIC FROM MADNESS" THEME, AS IT KEEPS POPPING UP IN THE STORY. IT'S NOT THAT I DON'T THINK THAT IT'S INTERESTING TO *ALSO* LOOK AT THAT SIDE...

I THINK IT'S THE *CENTRAL* ISSUE!

WHY SHOULD IT BE?

LISTEN: FORGET FOR A MOMENT THESE ARE HISTORICAL PEOPLE, INTENT ON BUILDING FOUNDATIONS FOR MATHEMATICS...

...AND THINK OF THE "GUEST" AS A FICTIONAL STORY...

AS *WRITER*, WOULD YOU SAY IT'S MORE MOTIVATED BY CHARACTER OR ACTION?

BIT OF BOTH, I THINK.

TO ME IT'S 100% CHARACTER! NOT JUST THEIR ACTIONS BUT THEIR *IDEAS* COME FROM IT: ONLY MEN LIKE *THEM* COULD HAVE THOUGHT THEM!

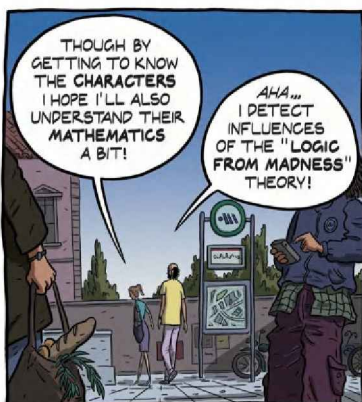
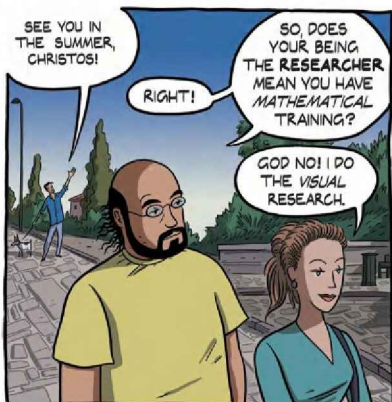
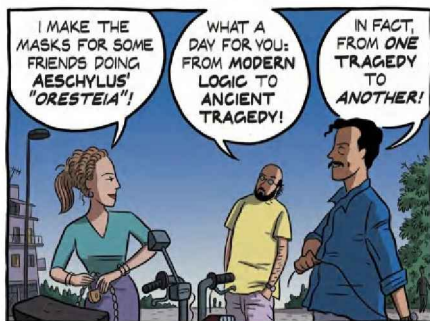
YOU MEAN IF THEY WEREN'T NEUROTIC, OR WHATEVER, THEY WOULDN'T HAVE THE NECESSARY PASSION AND PERSISTENCE TO CREATE LOGIC?

...OR THAT THE *IDEAS THEMSELVES* WERE INSPIRED BY NEUROSIS?

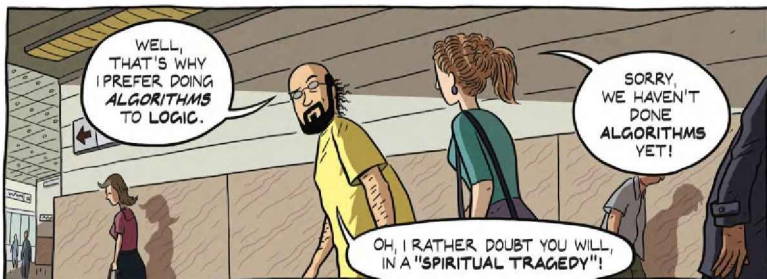
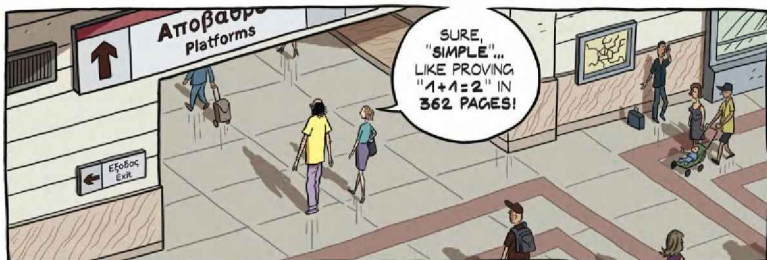
REMEMBER BERTIE TO ERIC ON "1+1=2 IN 362 PAGES"?

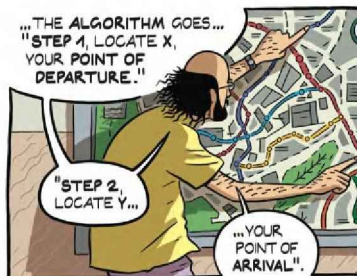
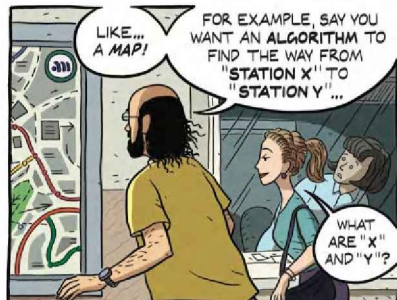
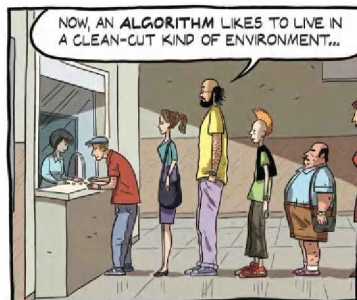
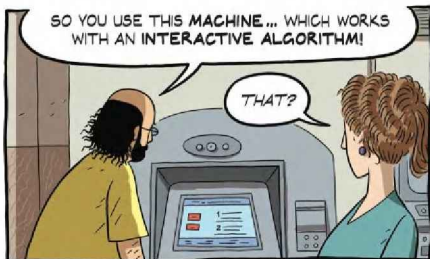
YEP! HE SAID SOMETHING LIKE "THE PRICE YOU PAY FOR ABSOLUTE CERTAINTY!"

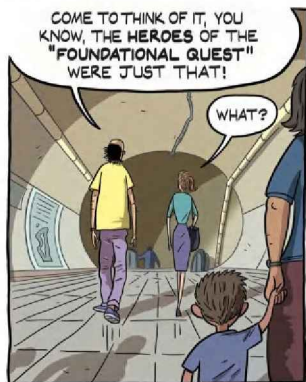
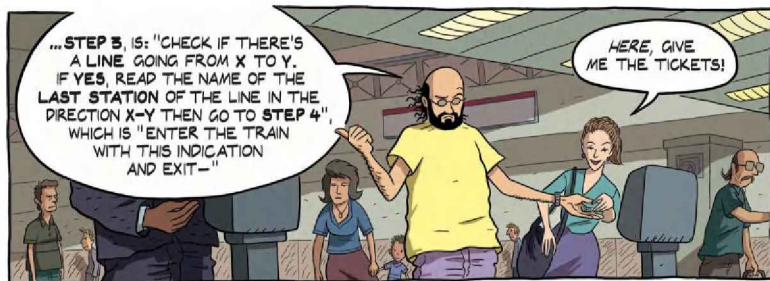
I GUESS MY POINT IS THAT *LESS TORTURED* CHARACTERS WOULD NOT HAVE FOUND THIS PRICE WORTH PAYING!











Hello Friend,

I arrived at Berkeley this morning...

But I've been thinking of Athens.

Of the "Foundational Quest in Comics"...

...To which I've given the nickname "Logicomix"...

...And its meaning.

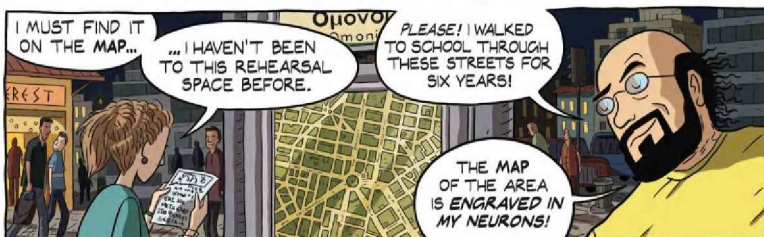
So I want to tell you a little story.

Which I think brought me a bit closer...

...To the "Logic and Madness" theme.

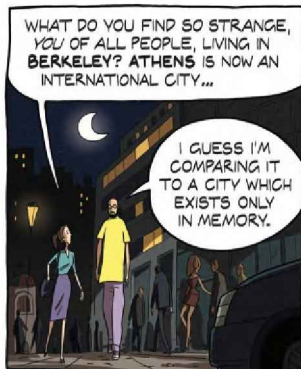


... An Athens story.









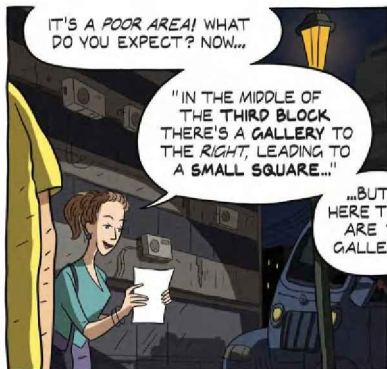
WHAT DO YOU FIND SO STRANGE, YOU OF ALL PEOPLE, LIVING IN BERKELEY? ATHENS IS NOW AN INTERNATIONAL CITY...

I GUESS I'M COMPARING IT TO A CITY WHICH EXISTS ONLY IN MEMORY.



YOU KNOW, AT LEAST 10% OF THE POPULATION OF GREECE IS NOW IMMIGRANT.

AROUND HERE IT'S PROBABLY MORE LIKE 99%!



IT'S A POOR AREA! WHAT DO YOU EXPECT? NOW...

"IN THE MIDDLE OF THE THIRD BLOCK, THERE'S A GALLERY TO THE RIGHT, LEADING TO A SMALL SQUARE..."

...BUT HERE THERE ARE TWO GALLERIES!



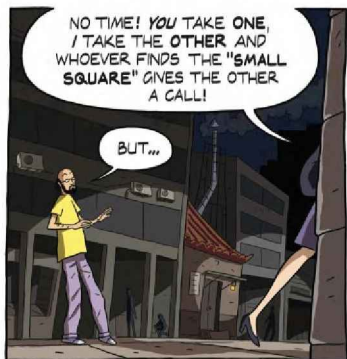
SO WHAT DOES YOUR "ENGRAVED MAP" TELL YOU NOW?

THAT THESE INSTRUCTIONS WERE WRITTEN BY A PERSON NOT TRAINED IN ALGORITHMIC EXACTNESS!



LET ME SEE... WE'LL APPLY A SIMPLE SEARCH PROCEDURE: FIRST, TAKE THE ONE GALLERY, AND IF IT DOESN'T LEAD WHERE WE WANT IT TO...

...WE CONCLUDE BY THE "REDUCTIO AD ABSURDUM" WE MUST TAKE THE OTHER!



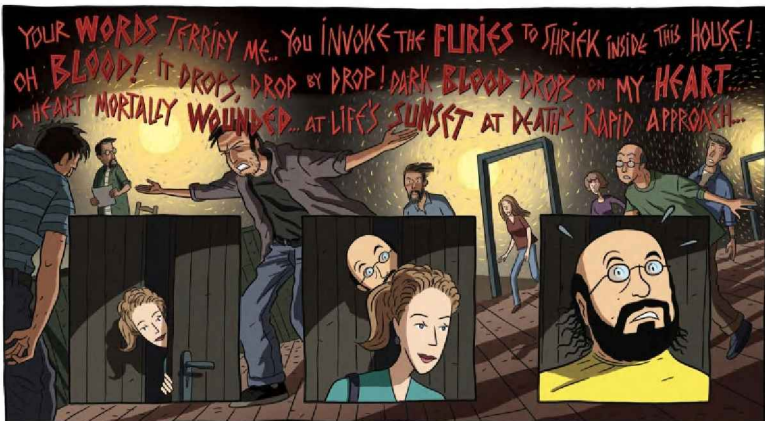
NO TIME! YOU TAKE ONE, I TAKE THE OTHER AND WHOEVER FINDS THE "SMALL SQUARE" GIVES THE OTHER A CALL!

BUT...











Strangely, the "Oresteia"'s conflicts brought to mind...

...My own puny little hubris of earlier that evening...

...My thinking that I knew an area of Athens...

...Just because I had, as I said, "its wap engraved in my neurons."

Good grief!

And then, strangely, this brought back my earlier comment to Anne on "wap-makers"...

...And the heroes of this "Logicmix" we're trying to make.

And I thought: "Sure, Frege, Russell, Whitehead, were excellent wap-makers..."

"... But maybe eventually they confused their reality with their waps."

WHAT A PERFECT DEFINITION OF INSANITY!

...And I felt this idea gives your theme of "Logic from Madness" a form I understand.

## 5. LOGICO-PHILOSOPHICAL WARS



SO, CHRISTOS'S COMMENT ABOUT MAPS vs. REALITY COMES AT A TIMELY MOMENT IN OUR STORY, JUST AS THE REAL WORLD BEGINS TO *BARGE* INTO RUSSELL'S CLOISTERED LIFE.



Sitting on Brighton beach one wintry day, my wind went back to my early years...



...The time when Euclid saved me from the clutches of Grandmother's stern religion.



The promise of certainty in total rationality was my dream of a perfect cosmos.



My own vision of Heaven.



All in all, I'd spent twenty years struggling with the Foundations of Mathematics...

...My own idealistic foray into the great Ocean of Truth.

And now the time had come...

The "*Principia Mathematica*" was about to be published, bringing my labours to the world...



...Or, to be exact, to the tiny part of the world that could understand it.

This sense of an ending, however incomplete, prompted me to review my life until then.

And the review brought home an unwelcome truth:

Apart from Mathematics, and a bungled attempt at marriage, I was totally isolated!

Here I was ...

A fish in a bowl...

Cut off from the world...



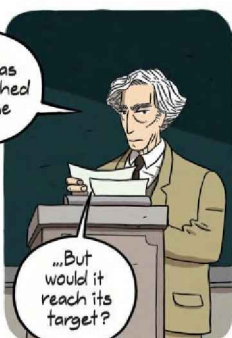
...Content to chop at the parts of it that could fit through the grid protecting my austere intellectual lair.





I realized then that, at the human level, I hadn't progressed much from the sad little boy desperately seeking ways out of the deadly vortex of uncertainty.

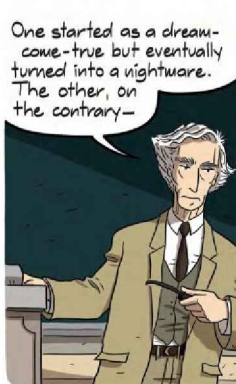
The "Principia" was my outstretched hand to the world...



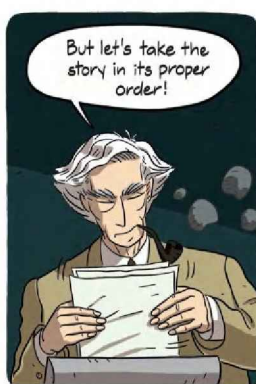
...But would it reach its target?



So! We are now in the year 1911. In it, two events of momentous import occurred...



One started as a dream-come-true but eventually turned into a nightmare. The other, on the contrary—

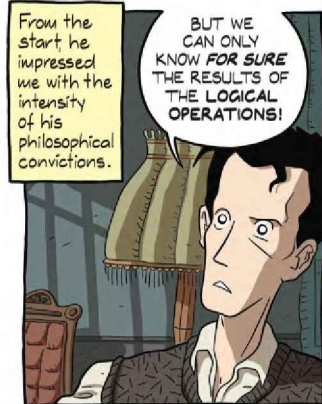


But let's take the story in its proper order!



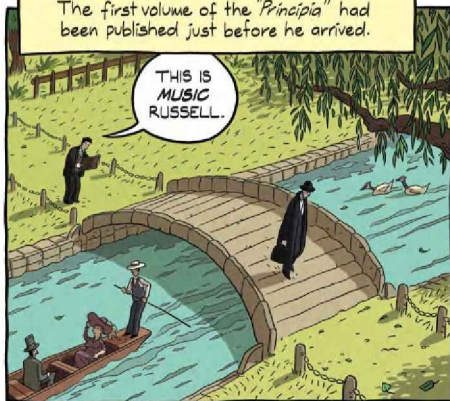
I was sitting in my rooms at College, one afternoon. There was a knock at the door...

YOU ARE THE HERR PROFESSOR RUSSELL?



The first volume of the "Principia" had been published just before he arrived.

THIS IS MUSIC RUSSELL.



THIS IS MOZART!

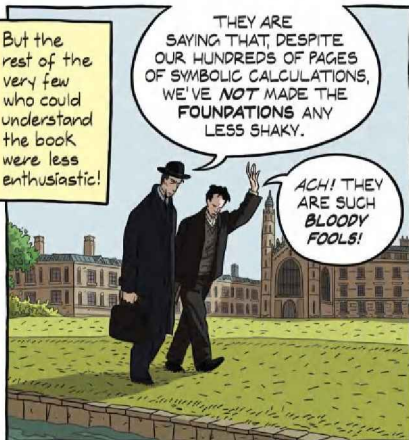
Wittgenstein's aesthetic appreciation was, of course, heartwarming...



But the rest of the very few who could understand the book were less enthusiastic!

THEY ARE SAYING THAT, DESPITE OUR HUNDREDS OF PAGES OF SYMBOLIC CALCULATIONS, WE'VE NOT MADE THE FOUNDATIONS ANY LESS SHAKY.

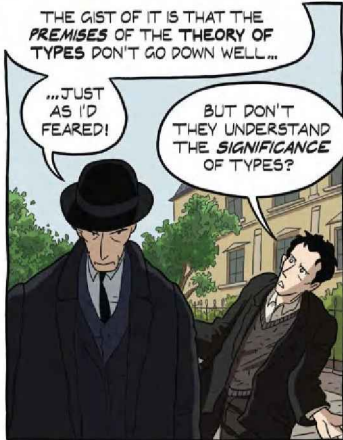
ACH! THEY ARE SUCH BLOODY FOOLS!



THE GIST OF IT IS THAT THE PREMISES OF THE THEORY OF TYPES DON'T GO DOWN WELL...

...JUST AS I'D FEARED!

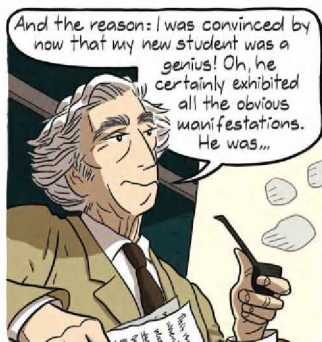
BUT DON'T THEY UNDERSTAND THE SIGNIFICANCE OF TYPES?

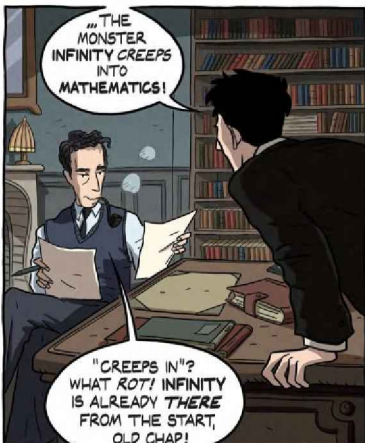
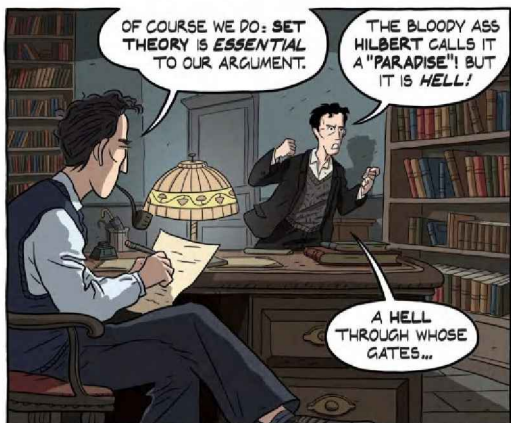
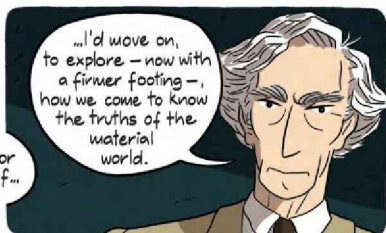
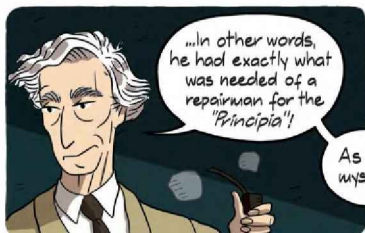


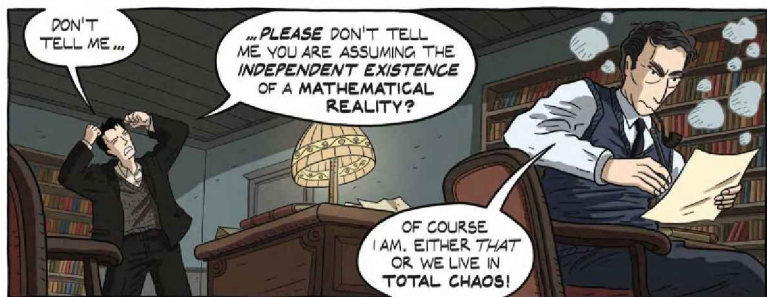
THEY ARE OUR SAFEGUARD AGAINST PARADOX, THEY ARE ESSENTIAL TO LOGIC ITSELF! TYPES MUST BE SALVAGED...

...AT ALL COSTS!









DON'T TELL ME ...

...PLEASE DON'T TELL ME YOU ARE ASSUMING THE INDEPENDENT EXISTENCE OF A MATHEMATICAL REALITY?

OF COURSE I AM. EITHER THAT OR WE LIVE IN TOTAL CHAOS!



BUT THE MATESTY OF THE "PRINCIPIA" IS THAT IT PUTS ON PAPER EVERYTHING AND ONLY WHAT IT SAYS!

SO WHERE IS THIS "INFINITY" OF YOURS?



WHERE???

IT CAN'T FIT...



...IN A FINITE BOOK!!!

When I'd assigned Wittgenstein to fine-tune our technical arguments...

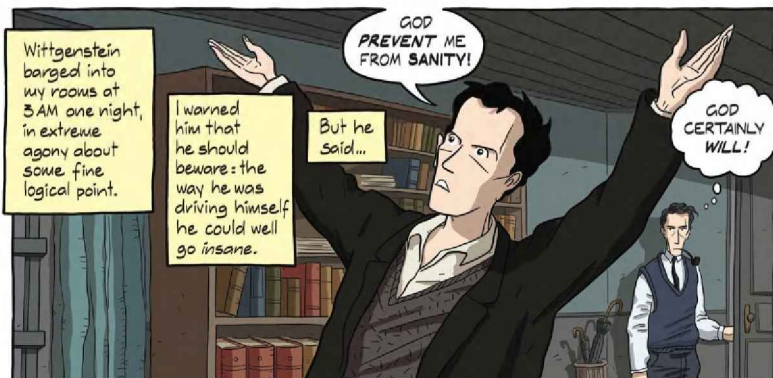
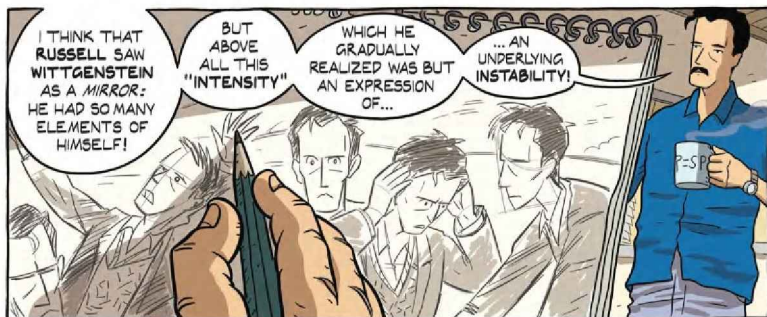
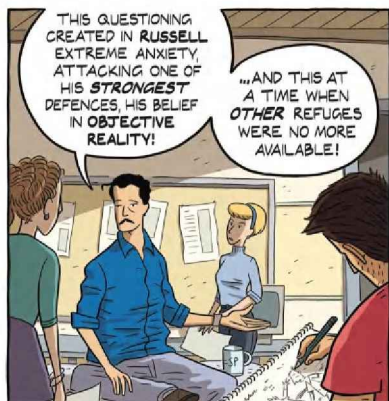


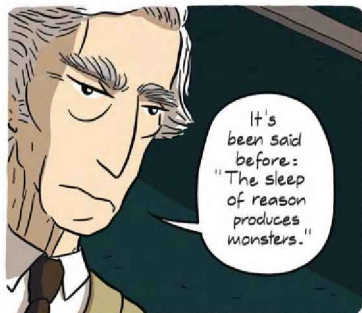
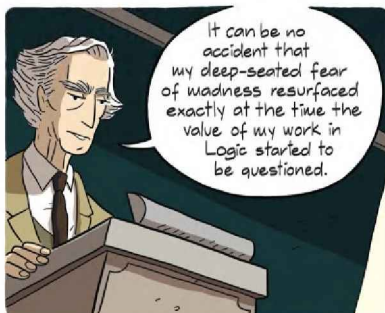
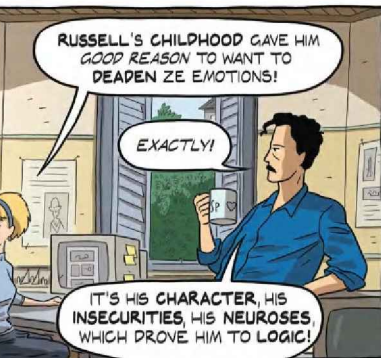
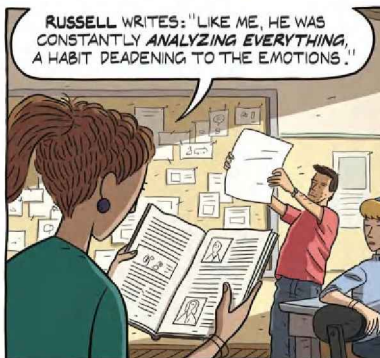
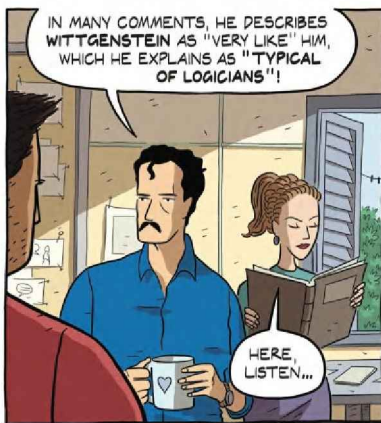
...I'd hoped, naively, that he would do precisely as asked.

SIGH

DON'T YOU SEE... BLAH BLAH ...NO OBJECTIVE EXISTENCE BLAH BLAH

But now he was questioning my most basic, unspoken premises about the nature of Truth!







Being a true child of the Victorian age, I had learned to regard every human being as essentially split into two persons.



The one a paragon of purity,  
whose credo was Reason...

...But the other  
a disgusting  
reprobate,  
always seeking  
unrestrained  
voluptuousness!

It was he who Wittgenstein  
had brought to the surface, by  
undermining my logical work.

I was driven into a ghostly cul-de-sac.



COME WITH ME!  
LET'S HAVE SOME  
FUN!

"THOU SHALT  
**NOT** LIVE HAPPILY  
UNTIL THOU SOLVEST  
THE PROBLEMS OF  
THE THEORY OF  
TYPES!"

It was then that the second momentous  
event occurred.

During this crisis, I was clinging for some kind of support to the external routines of academic life.

And it was during a boring dinner at College that it started...





In Evelyn's eyes, I'd seen  
the stark, unvarnished  
image of our predicament...

The tragic loneliness  
of every human being.

The finiteness and  
profound futility of life.

The terrible harshness  
of pain and disease.

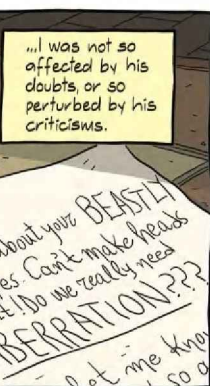
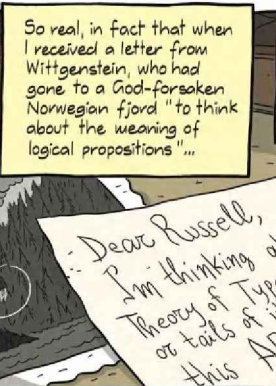
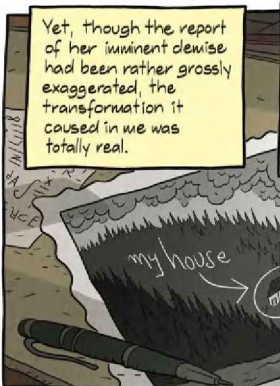
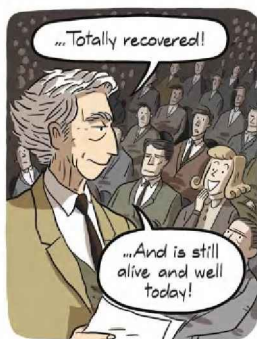
The  
unmitigated  
horror of  
death.

But talking to Eric  
I understood there is  
also an alternative:

Redemption.

In compassion...

In love...



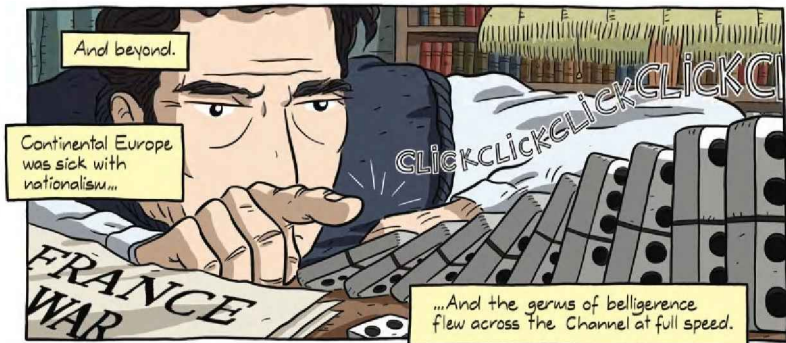
You see, we have now reached the summer of 1914.

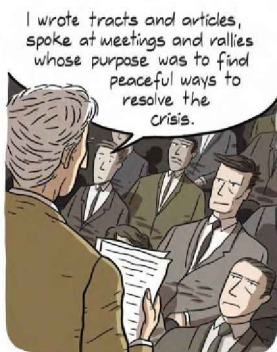
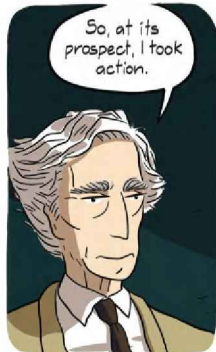
# Pathé gazette

AUSTRIAN ARCHDUKE ASSASSINATED!

OH DEAR!











On August 4, 1914, I was attending a pro-peace rally at Trafalgar Square.

It was there that the news reached us...

...The United Kingdom had just declared war on Germany!



And then, I witnessed a terrifying miracle.

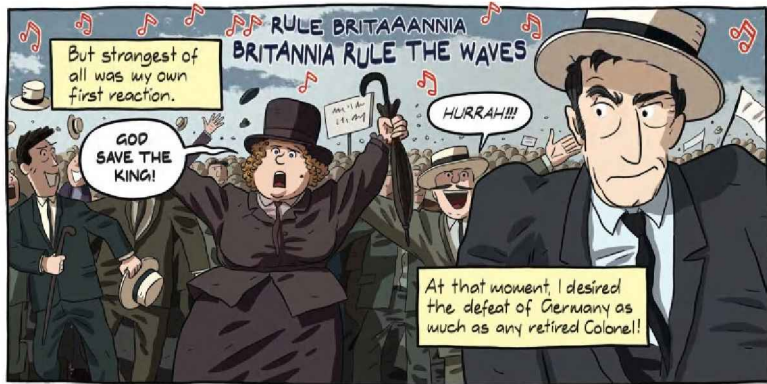


People who were there united by a dream of peace...



GET THE HUNS!

...Were, suddenly, feasting the new reality of war!



But strangest of all was my own first reaction.

GOD SAVE THE KING!

RULE BRITANNIA  
BRITANNIA RULE THE WAVES

HURRAH!!!

At that moment, I desired the defeat of Germany as much as any retired Colonel!

Thankfully, this strange upsurge of my deep-buried, tribal instincts lasted but a few hours. Then reason took over again.



I started, in my lectures and articles, to argue against the madness that was engulfing some of the cleverest people I knew, including...



...Ludwig Wittgenstein!

My own "Crown Prince of Logic" had now enlisted, as a volunteer in the Austro-Hungarian Imperial Army!



Although Wittgenstein had his own, eccentric reasons for becoming a soldier...

...The military powers-that-be, no doubt influenced by his family's immense wealth, put him at Corps Headquarters.



WE WANT BIG CANNON, WITTGENSTEIN: THE GENERAL IS A BIT SHORT-SIGHTED!

JAWOHL, HERR SERGEANT!

He was now an Assistant Mechanic, no less!



Yet no occupation could interrupt his latest train of thought, about the meaning of logical propositions...

...And their relations to language.

NOW THAT IS INTERESTING!



In the first years of the war, he got some letters through to me...



...Describing his latest efforts.



It was toy models that led him to his first big idea.



...SO OUR ARTILLERY WILL COMMENCE FIRING ON THE ENEMY'S WESTERN FLANK.

EACH CANNON STANDS FOR ONE BATTERY MEINE HERREN.

A TOY SOLDIER REPRESENTS AN INFANTRY BATTALION.

SO, EACH PART OF REALITY IS REPRESENTED BY A SYMBOL!

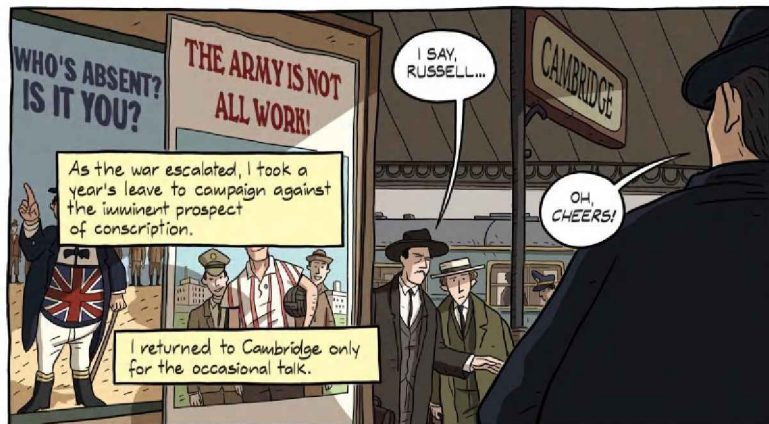
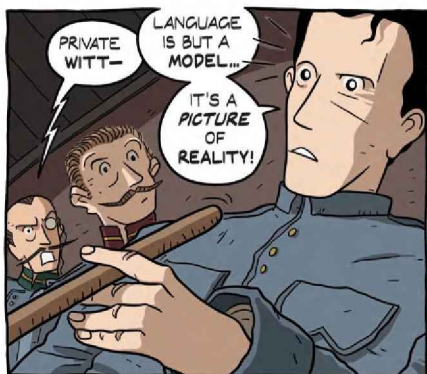
...AND THEN THE FOUR INFANTRY BRIGADES WILL ATTACK THEIR FRONT.



BRILLIANT!

IN GOES THE INFANTRY...

...AND THESE RECOMBINE, IN ACCORDANCE WITH THEIR REAL RELATIONS AS MEDIATED BY LANGUAGE!







In a sense, Wittgenstein had volunteered for the same reason, though his sense of "fun" was philosophically biased!

HERR MAJOR, I HAVE A PETITION CONCERNING MY TRANSFER TO THE FRONT!

YOU ARE NOT THE ONLY ONE.

HAVE YOU GOT CHILDREN?

NO, HERR MAJOR.

BUT YOU ARE MARRIED?

I AM NOT.

YOU DON'T UNDERSTAND! YOU MUST HAVE **VERY SERIOUS REASONS** NOT TO GO TO THE FRONT.

WITH RESPECT, SIR, IT IS **YOU** WHO DOES NOT UNDERSTAND.

MY PETITION IS TO GO TO THE FRONT!

Wittgenstein believed that before being a logician he "should become a human being".

And so he took Schopenhauer's word for it: there is nothing like a good near-death experience, to humanize you!

I SHALL RECOMMEND YOU.

And recommend him he did.

Wittgenstein got his wish.

He was assigned to a front-bound artillery regiment.



At last he could enjoy the luxuries of trench life.



Relish in the comfort of sleeping in mud.



No task was too deweaning...

None too harsh for him...

Too arduous...

...Or too dangerous.

ANY VOLUNTEERS?

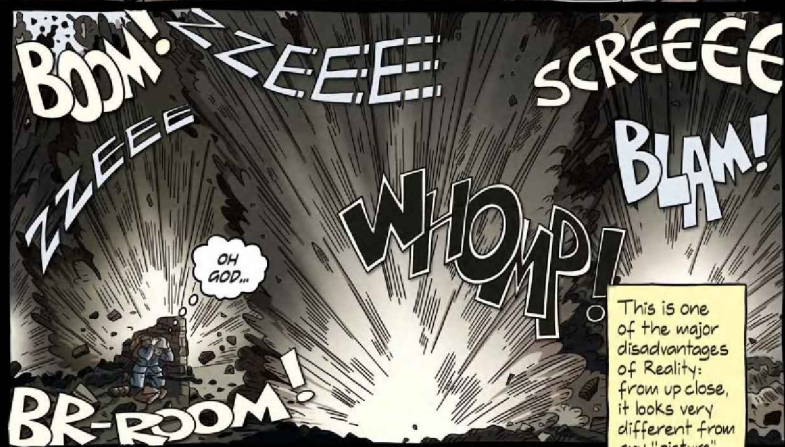
HOW ABOUT YOU?

OF COURSE, HERR STAFF SERGEANT!

Ah, the superior wasochism of the privileged!





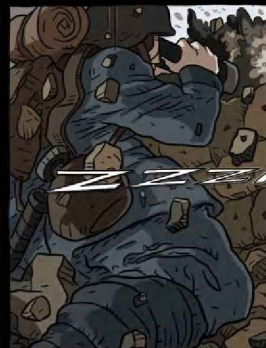


This is one of the major disadvantages of Reality: from up close, it looks very different from any "picture".

... A difference  
which no theory  
can explain.




SANG-FROID  
SANG-FROID...

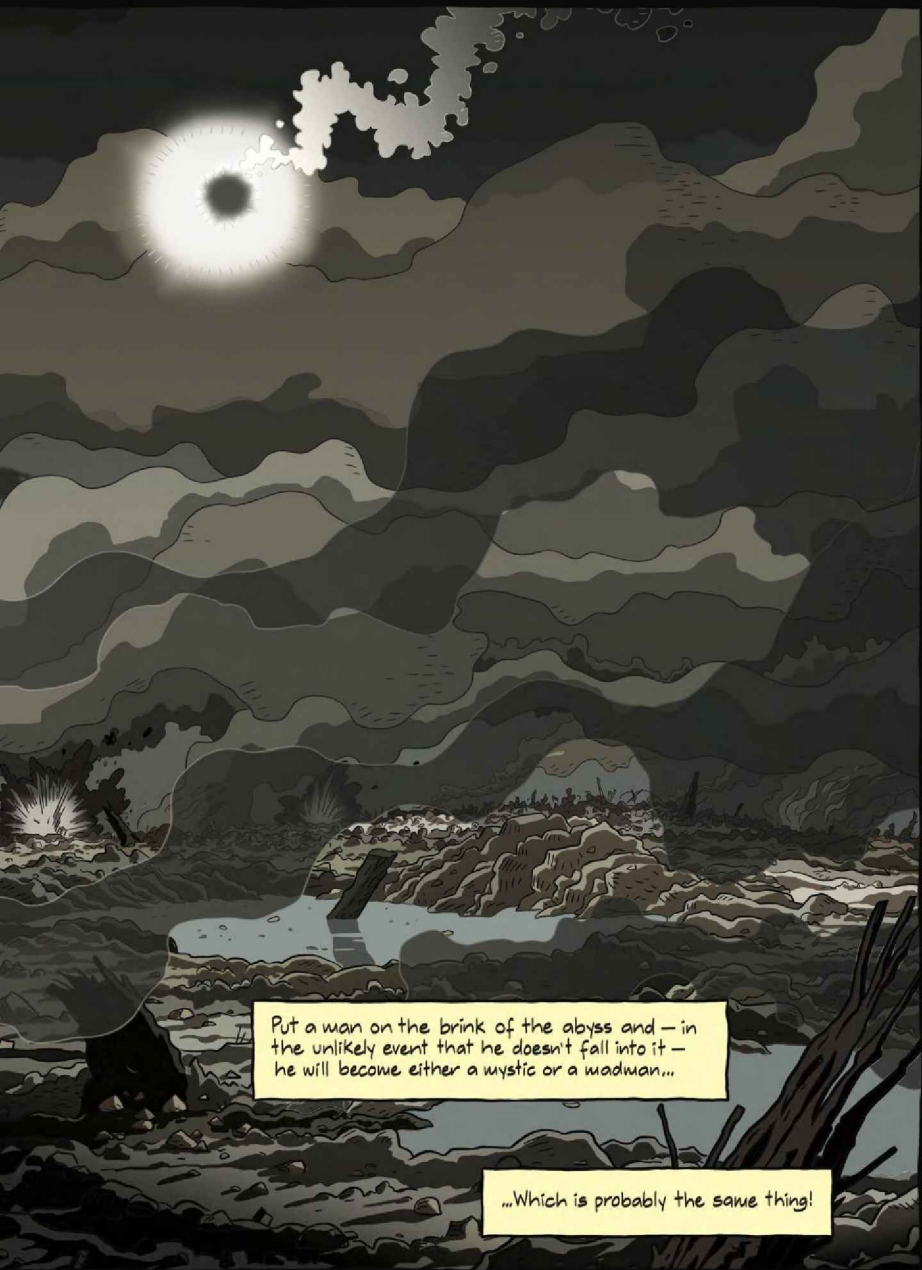


Face to face with death,  
Wittgenstein arrived at  
his fundamental insight.



A man in a blue jacket stands in a dark, desolate landscape. To his left is a stack of brown boxes. In the center, a small fire burns brightly. The background is filled with dark, jagged, mountain-like shapes under a heavy, dark sky. A thought bubble above the man contains the text: "THE MEANING OF THE WORLD DOES NOT RESIDE IN THE WORLD!".

THE MEANING  
OF THE WORLD  
DOES NOT RESIDE  
IN THE WORLD!

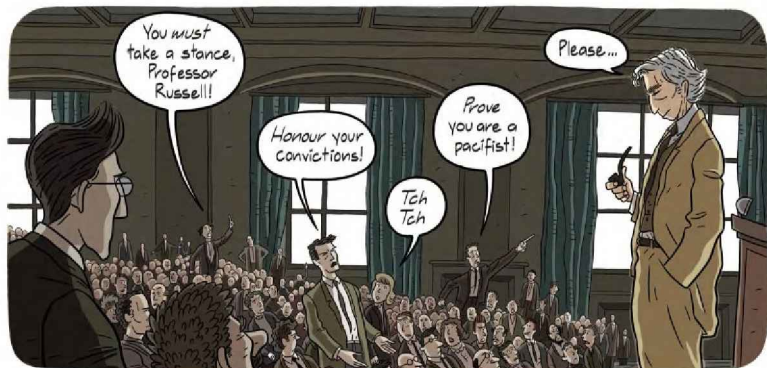


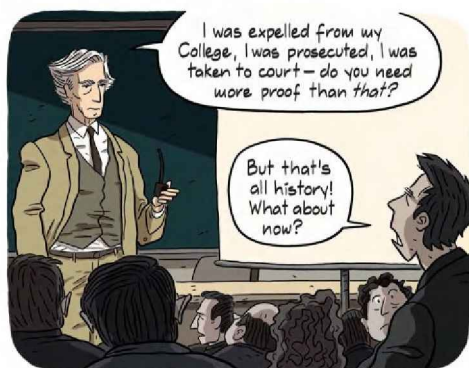
Put a man on the brink of the abyss and — in the unlikely event that he doesn't fall into it — he will become either a mystic or a madman...

...Which is probably the same thing!



...Which involved changing my tack: from arguing simply for a peaceful resolution of the conflict, I now urged people to become conscientious objectors.







A few months after the end of the war, I received a totally unexpected gift.

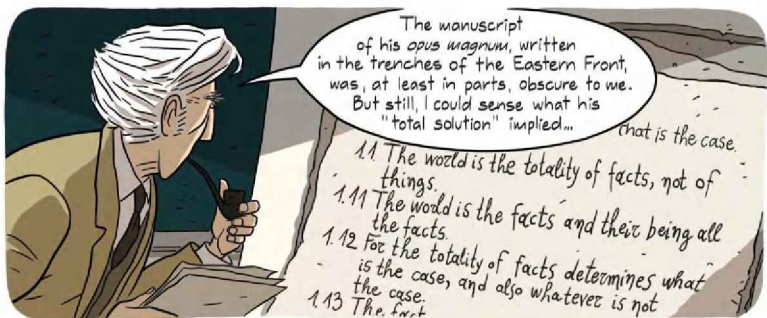


THANK GOD... HE IS ALIVE!



"I HAVE TOTALLY SOLVED ALL THE PROBLEMS OF PHILOSOPHY!"

Good old Wittgenstein... Modesty was never his strong point!



The manuscript of his opus magnum, written in the trenches of the Eastern Front, was, at least in parts, obscure to me. But still, I could sense what his "total solution" implied...

that is the case.

- 1.1 The world is the totality of facts, not of things.
- 1.11 The world is the facts and their being all the facts.
- 1.12 For the totality of facts determines what is the case, and also whatever is not the case.
- 1.13 The fact

WAIT, WAIT! I TRIED TO READ THIS "TRACTATUS LOGICO-PHILOSOPHICUS" ...



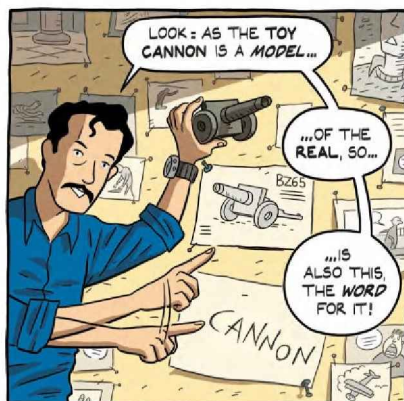
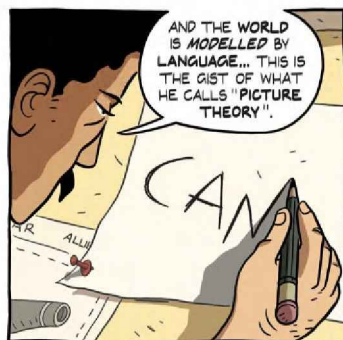
STILL CALLED, BACK THEN, "LOGISCHE-PHILOSOPHISCHE ABHANDLUNG".

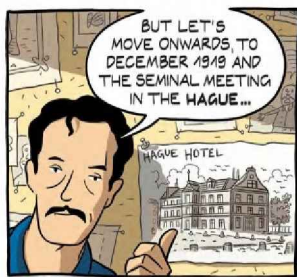
...WHATEVER, AND I GOT VERY CONFUSED!

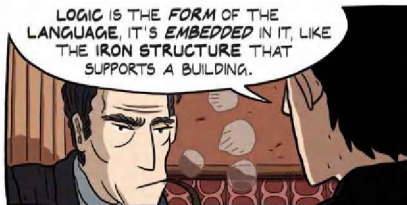


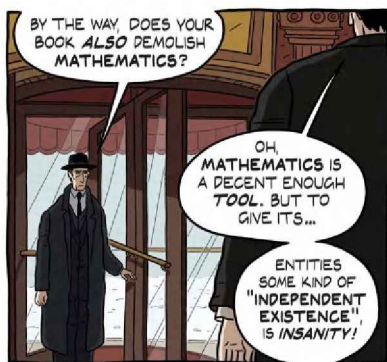
IN THAT, YOU ARE NOT ALONE.

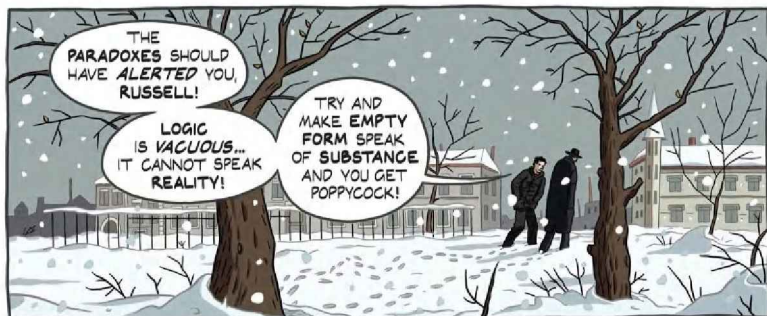
WHAT IS ZE "NUTSHELL" OF IT?

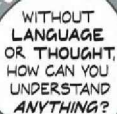








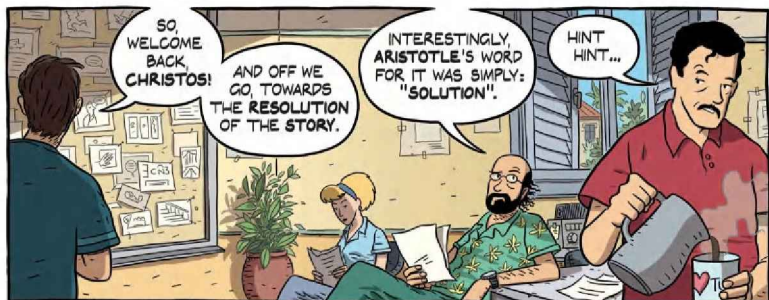


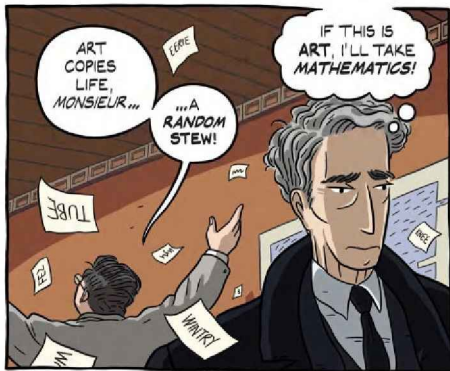
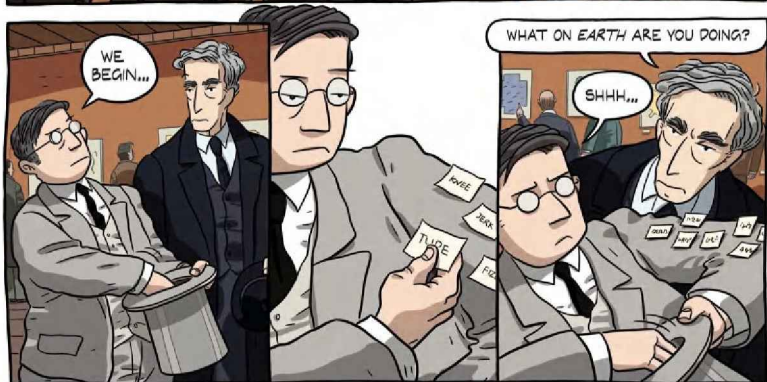


## 6. INCOMPLETENESS



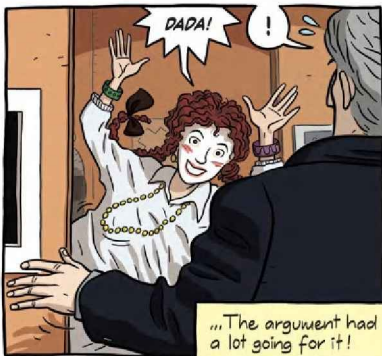






Prelude: the Old World had created a monstrous war.

Conclusion: The Old World's values and the art that embodied them should be destroyed.



...The argument had a lot going for it!



I was as critical of the Old World as the angriest artist.

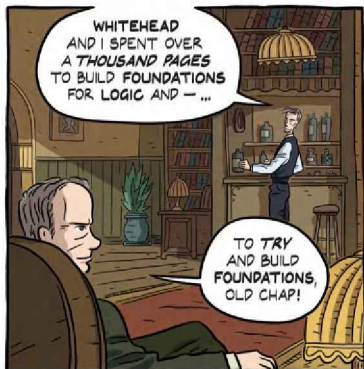
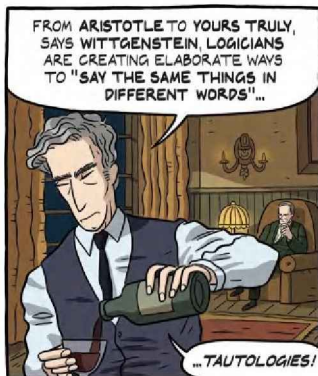
But I feared the void created by its demise...

...An open invitation to the Irrational.

W.B. Yeats's lines expressed my apprehension perfectly.



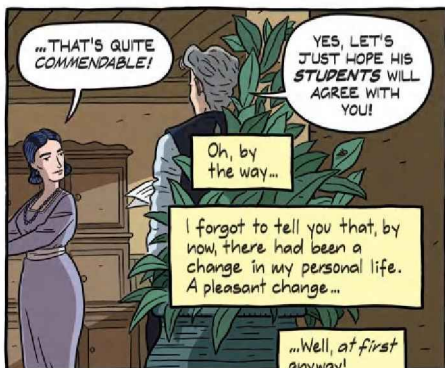
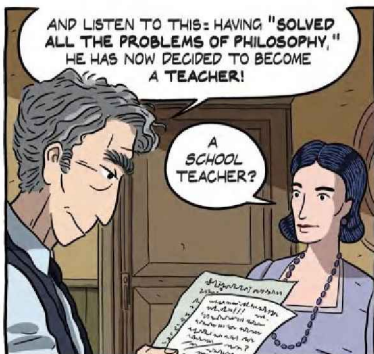
...Mere anarchy is loosed upon the world."



Yet, despite my doubts about Wittgenstein's Logic, I was full of admiration for his integrity.



THE MAN HAS SURPASSED HIMSELF IN ECCENTRICITY!



My new wife, Dora, shared my interest in the welfare of that most indiscriminating of clubs: Humankind.

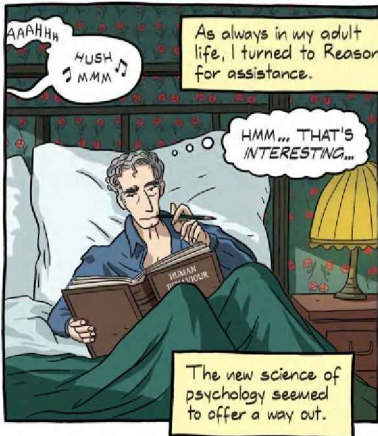
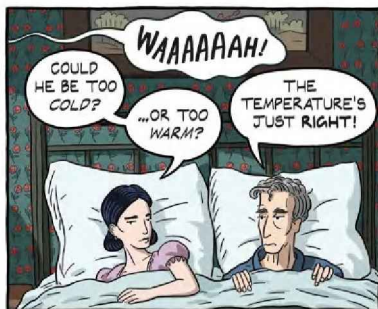
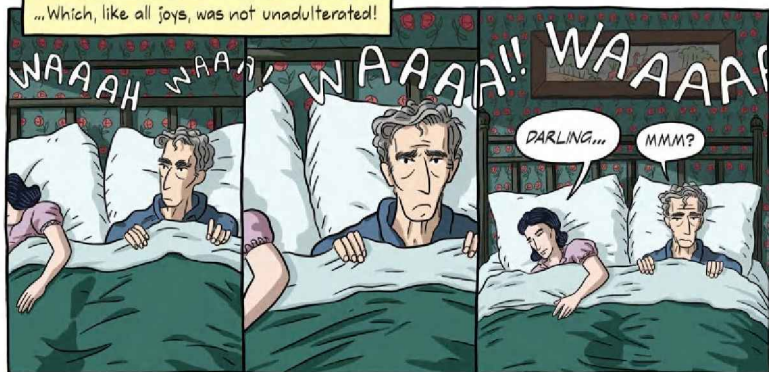
... To which, incidentally, a new member was about to be added.



Philosophy's consolations had not prepared me for such joy.



...Which, like all joys, was not unadulterated!



In fact, the time seemed propitious for an extension of my logicist project.



A group of visionaries in Vienna had drafted a manifesto advocating "the scientific conception of the world"...

... A project to apply the tools of Logic, Mathematics and the Physical Sciences to the study of human matters.



So, despite the failure of my own work in Logic, I did not—

WAIT!



YOU JUST CAN'T GO CALLING RUSSELL'S WORK IN LOGIC A "FAILURE"...



...NO WAY!

IT'S HIS WORDS WE ARE USING!

BUT THE "PRINCIPIA" IS THE BASIS OF EVERYTHING THAT FOLLOWED!

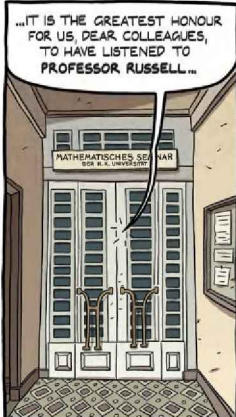


BUT—

MMM...

OK, MAYBE WE'LL PUT SOME OF THAT IN BERTIE'S TRIP TO VIENNA!

...IT IS THE GREATEST HONOUR FOR US, DEAR COLLEAGUES, TO HAVE LISTENED TO PROFESSOR RUSSELL...



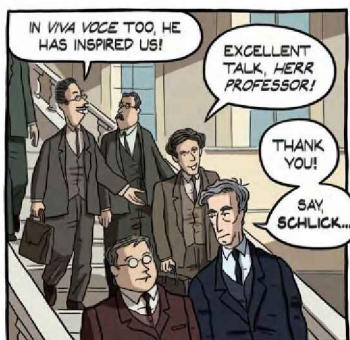
...WHO LAID THE GROUND FOR A LOGICAL LANGUAGE MAKING POSSIBLE THE SCIENTIFIC WORLD-VIEW! IT IS HIS PIONEERING VISION WHICH INSPIRES THE WORK OF OUR CIRCLE...

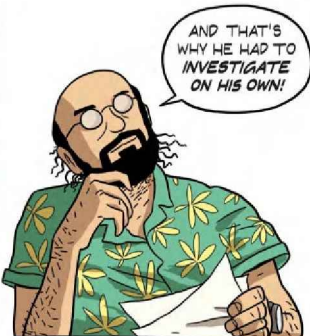
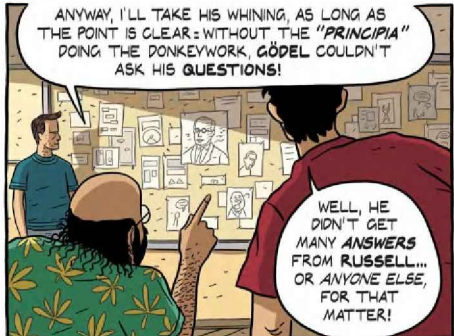
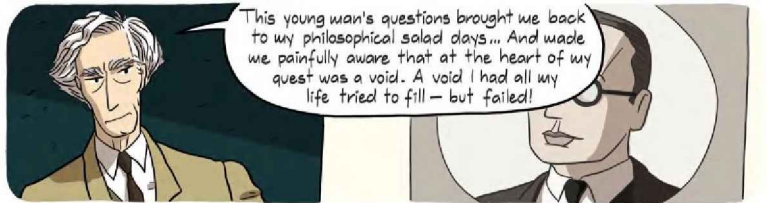
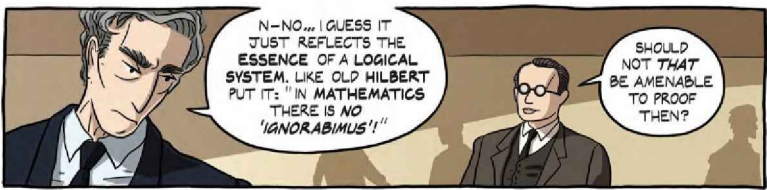


TOGETHER WITH THAT OF THOSE VENERABLE OLD GENTLEMEN, FREGE AND WITTGENSTEIN!

?



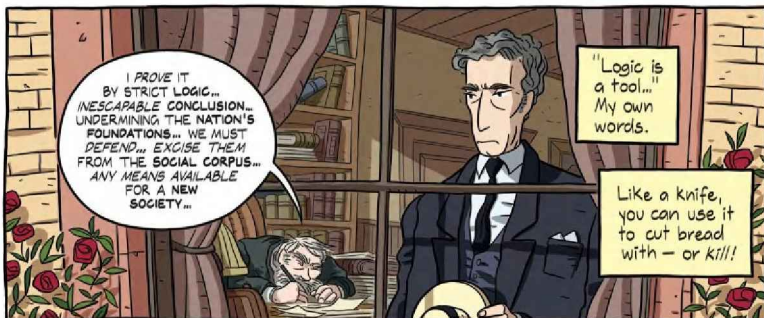




I set out for home, rejuvenated by the optimism of the Vienna Circle.

...Making a stop to visit an old friend.

FRAU FREGE?



Frege's paranoid vision played on a malignant variation of an ancient theme:

"You can't make a good omelet with bad eggs."

HELP!  
HE-EEELP  
ME!

ARE  
YOU SURE HE'S  
ALRIGHT, MY  
LOVE?

MMM

But though I strongly disagreed with his racist criteria of excellence...

...Like him, I also dreamed of a better world...

HEL...  
GURGLE...  
HELP...

HE'LL BE FINE...  
...AS SOON AS HE  
OVERCOMES HIS  
PANIC.

...I'M HERE,  
LADDIE!

HELP!

WHAT  
THE-

SOB  
MOMMY...  
SOB

WHY ON  
EARTH DID YOU  
DO THAT?

?

HE WOULD'VE  
MANAGED  
PERFECTLY...

AND  
LEARNED  
SOMETHING  
FROM IT!

SORRY, GUV!  
IF I SEE 'IM DROWNING  
AGAIN, I'LL LET 'IM!

It's the oldest story around:  
Instinct, Emotion and Habit  
get the better of human beings.

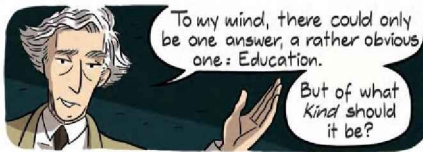
Or, in other words: start from the wrong premises and Logic can be the executioner's handmaiden — as in Frege's cruel theories. Or, alternatively, a fool's ideal accomplice!



So how to straighten "the crooked timber of humanity"? How to annul the harm done by... Instinct, Emotion and Habit?

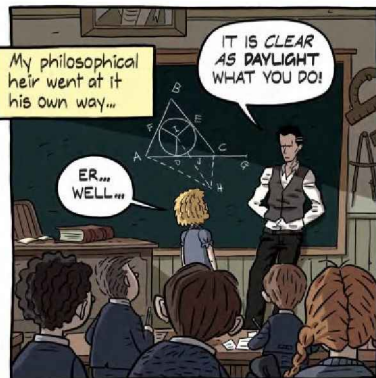


To my mind, there could only be one answer, a rather obvious one: Education.



But of what kind should it be?

My philosophical heir went at it his own way...



ER... WELL...

IT IS CLEAR AS DAYLIGHT WHAT YOU DO!

YOU KNOW THE TOOLS: COMPASS AND RULER! SO, TELL ME!



ER... I... DRAW A LINE FROM ANGLE "B".

ACH, "A LINE", JA! BUT WHICH LINE?



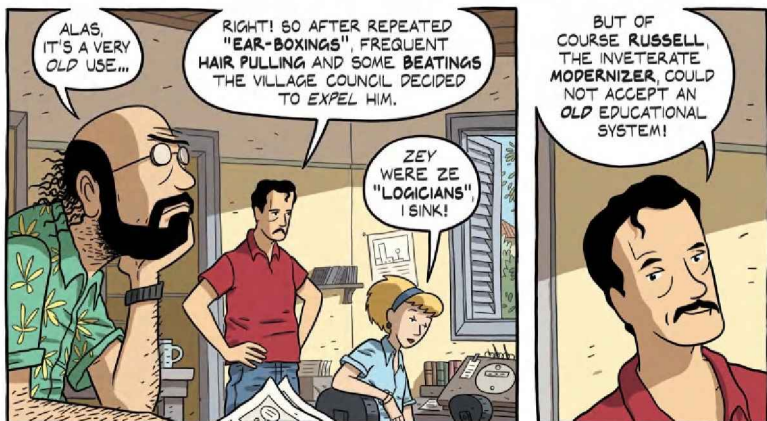
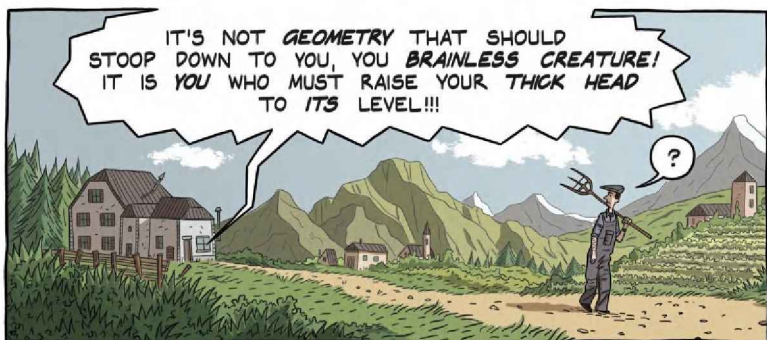
THE... HE-HEIGHT...

NOOOO! I'VE SAID IT A THOUSAND AND ONE TIMES!



WHICH LINE???

EM... ER... HERR WITTGENSTEIN THE... THE...



By now, I was convinced that the advances in scientific psychology offered a way out...

"The way to a perfect transformer, of bad eggs into good!"

WELCOME TO BEACON HILL

DEAR FRIENDS, WE SHALL NOW INTRODUCE YOU TO...  
...TOMORROW'S SCHOOL!

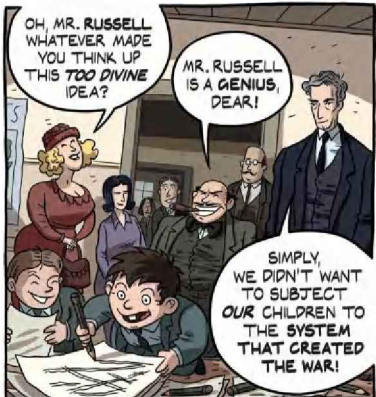
CLAP CLAP CLAP CLAP



...AND THIS IS THE ART ROOM, WHERE...

GEE!

LOOK, WILBUR! CRAYONS!



OH, MR. RUSSELL WHATEVER MADE YOU THINK UP THIS TOO DIVINE IDEA?

MR. RUSSELL IS A GENIUS, DEAR!

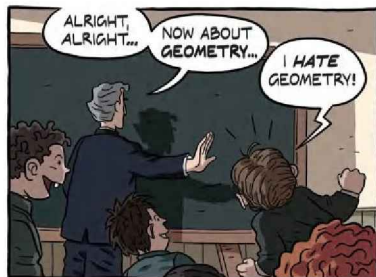
SIMPLY, WE DIDN'T WANT TO SUBJECT OUR CHILDREN TO THE SYSTEM THAT CREATED THE WAR!

In our brand-new school, Dora and I shared the tasks.

TODAY WE BEGIN WORK ON GEOMETRY WHOSE ACQUAINTANCE I MADE, LIKE YOU, WHEN I WAS VERY YOUNG...

OH, I THOUGHT YOU WERE ALWAYS OLD!

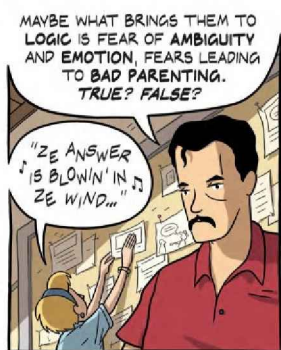


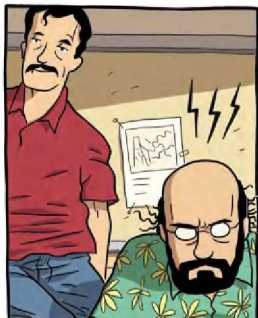
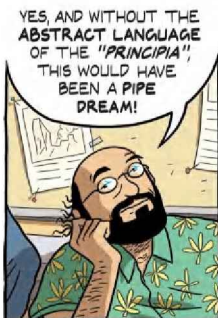
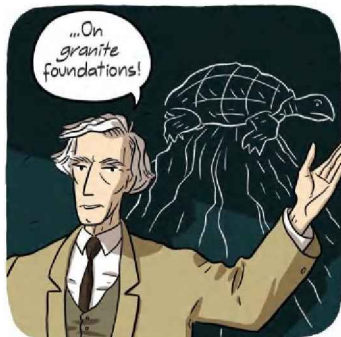


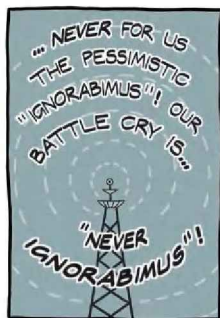




\*Zey are crazy zese logicians!

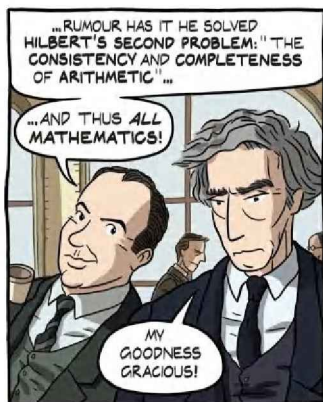
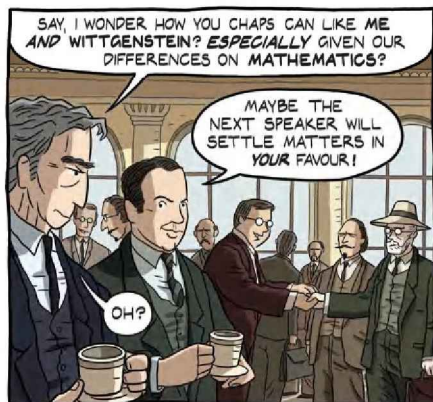
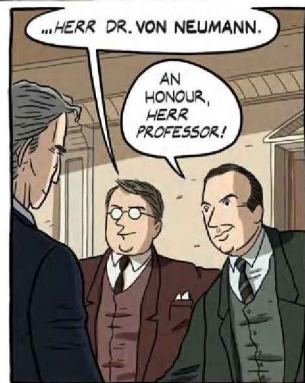
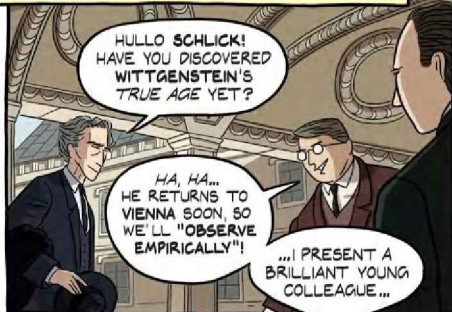






His message had inspired, among others, a recent acquaintance of mine, who was to be...

...A speaker at the next logical conference, held right inside the lair of the Vienna Circle.



It's hard to exaggerate the feeling of excited anticipation as Kurt Gödel began his talk.

HERR PROFESSORS HILBERT AND RUSSELL, DISTINGUISHED COLLEAGUES, I WILL SPEAK TO YOU OF...

HE'S A PLATONIST, LIGHT YEARS FROM WITTGENSTEIN!

...MY RESEARCH ON THE PROVABILITY OF THE PROPOSITIONS OF ARITHMETIC.

LIKE YOU, HE BELIEVES LOGIC IS AN IMAGE OF THE HIGHEST FORM OF TRUTH!

THE POWERFUL METHODS OF THE "PRINCIPIA" NOW ALLOW US, FOR THE FIRST TIME IN HISTORY, TO SPEAK OF A "CORRECTLY FORMULATED QUESTION" IN THEORIES OF MATHEMATICS...

...AND THIS ALSO, FURTHER, TO ASK: "IS A CORRECTLY FORMULATED MATHEMATICAL QUESTION NECESSARILY ANSWERABLE?"

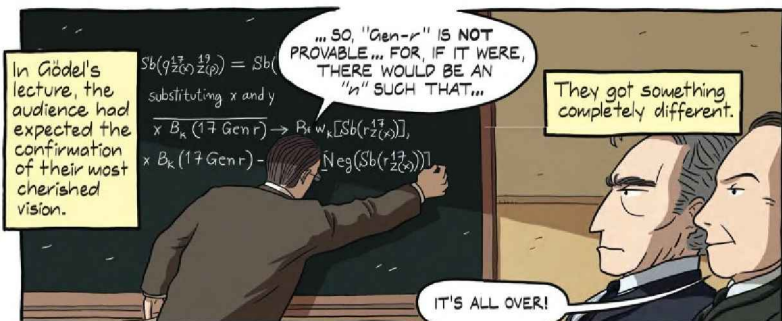
OBVIOUSLY!

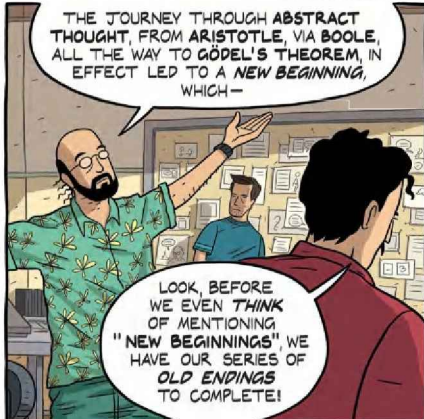
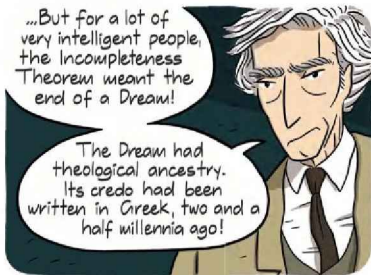
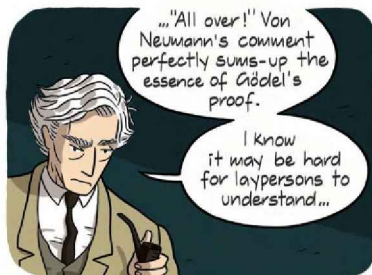
IN OTHER WORDS: "IS EVERY MATHEMATICAL STATEMENT PROVABLE, EITHER...

...THE STATEMENT ITSELF, OR - IF IT STATES SOMETHING FALSE - IT'S OPPOSITE?"

IT IS TO THIS, MOST FUNDAMENTAL QUESTION, THAT I HAVE FOUND THE ANSWER.

WHICH IS...





Now, as if Gödel's proof wasn't enough, my Viennese admirers soon received a new blow, which added insult to injury...

...Completely subverting, as it did, the image of my arch-rival for the Circle's admiration.

...TO CELEBRATE OUR FIRST MEETING, WE OFFER YOU OUR "MANIFESTO OF THE SCIENTIFIC WORLDVIEW".

...WRITTEN IN THE WAKE OF YOUR "TRACTATUS"...

...INIMITABLY ENCAPSULATED IN ITS LAST LINE...

... "WHAT WE CANNOT SPEAK OF, WE MUST PASS OVER IN SILENCE."

WHERE "SPEAK" NATURALLY MEANS "SPEAK LOGICALLY!"

YOUR WORK GAVE US THE MEANS TO EXPEL RELIGION, METAPHYSICS, ETHICS, ETC. FROM RATIONAL DISCOURSE!

SINCE "WHAT CANNOT BE SPOKEN ABOUT LOGICALLY" IS, QUITE LITERALLY, NON-SENSE...

...AND, OBVIOUSLY, BENEATH THE DIGNITY OF SERIOUS MINDS!

JUST WAIT A MINUTE!

THE MEANING OF THE "TRACTATUS" HAS COMPLETELY ESCAPED YOU!

ITS POINT IS THE EXACT OPPOSITE:

THE THINGS THAT CANNOT BE TALKED ABOUT LOGICALLY...

...ARE THE ONLY ONES WHICH ARE TRULY IMPORTANT!!!





Though arguments with him always involved some amount of sound and fury, Wittgenstein thankfully never resorted to physical violence...

...At least not against his peers!

... Open the waaaay for the Brown Battalions... We raise the Swastika, the hope of many millions!



Oh, if only that were also true of the acolytes of Europe's newest avatar of Irrationality.



HEY! THERE'S A YIP!



GET THE DOG!

I'LL TEACH YOU A LESSON IN ARYAN CULTURE!

?

As Nazi influence rose, incidents like this became frequent in, ostensibly still democratic, Austria.

...And against these, even our greatest minds were totally helpless.

IT'LL BE MUCH WORSE NEXT TIME, YIP!



EXCUSE ME... BUT WHY DID —

OY... OY...



WHATSA MATTER, PANSY? WAS THE JEWBOY YOUR LOVER?

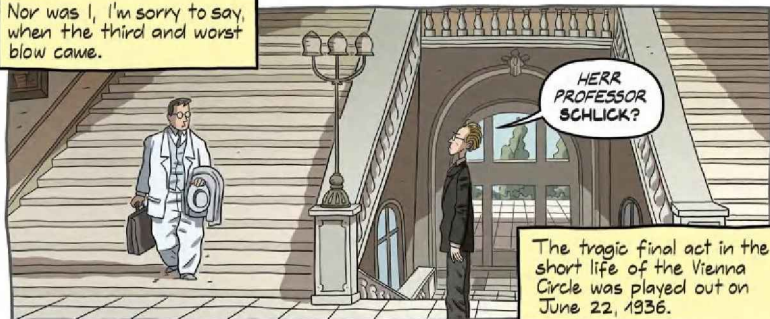
NO...

... NO...

In 1935, I learned Gödel was hospitalized for melancholia.

...I wasn't surprised.

Nor was I, I'm sorry to say, when the third and worst blow came.



HERR PROFESSOR SCHLICK?

The tragic final act in the short life of the Vienna Circle was played out on June 22, 1936.



I HAVE A PRESENT FOR YOU!



WHA—



BANG  
BANG  
BANG

The pro-Nazi newspapers declared that the Circle's rationalistic world-view had "desecrated hallowed Germanic values..."

...And thus Schlick deserved what he got.



The killer was a devotee of the new Teutonic god...

...Herr Adolf Hitler.

I want to express my gratitude to you, Ladies and Gentlemen, for your company on this, rather long, journey!



This journey from my earliest days to today, from Doubt to Certainty...



...And back again!

A journey of some joys and more disappointments, the latest of which is the realization that I've failed - also - as an educational reformer.



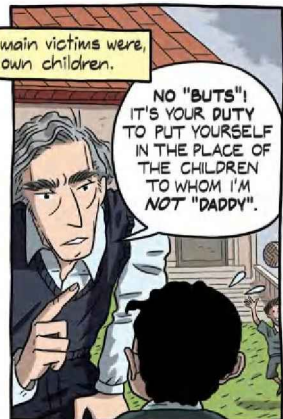
Here is a new, and much more bitter "Russell's Paradox"...



**YOU MUST REALIZE, JOHN THAT THIS IS YOUR SCHOOL, AND -**

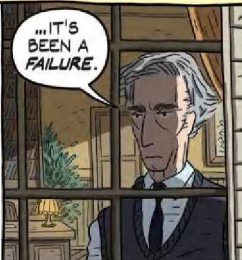
**BUT IT'S ALSO MY HOME, DAD AND -**

...Whose main victims were, alas, my own children.



**NO "BUTS"! IT'S YOUR DUTY TO PUT YOURSELF IN THE PLACE OF THE CHILDREN TO WHOM I'M NOT "DADDY".**

Dora and I had created Beacon Hill primarily to give our own children an ideal education.



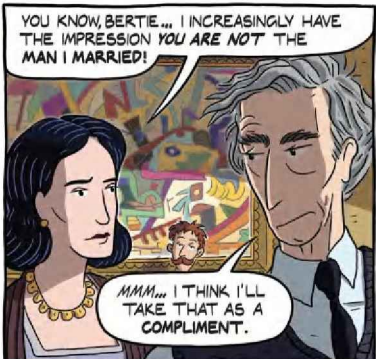
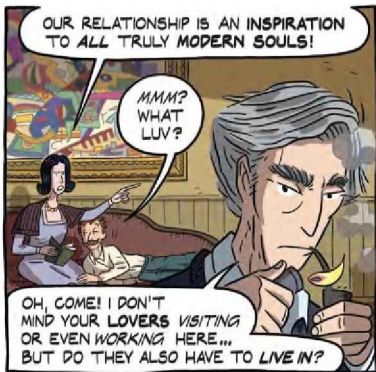
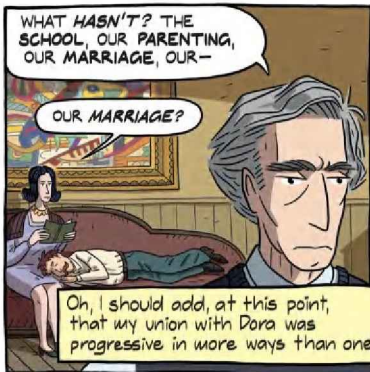
...IT'S BEEN A FAILURE.

We completely ignored the fact that, in the process, we deprived them of both home and parents!

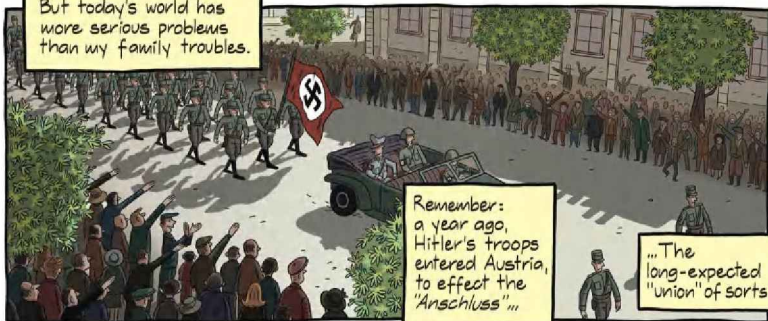


**BROOM**

**WHAT HAS, BERTIE?**



But today's world has more serious problems than my family troubles.



Remember: a year ago, Hitler's troops entered Austria, to effect the "Anschluss"...

...The long-expected "union" of sorts.



NOOO!

The Jews, as well as anyone dissenting from the Nazi ideology, have been violently rounded-up and sent off to destinations still unknown.

One of the first acts of the new, Nazi rulers was to release Schlick's murderer.

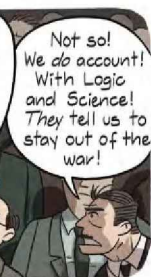
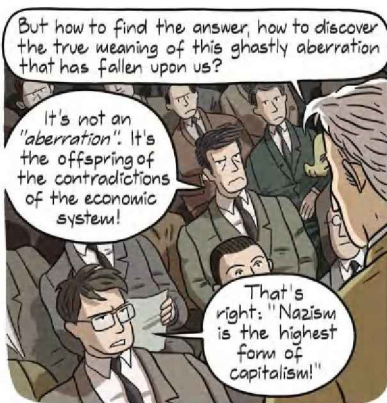


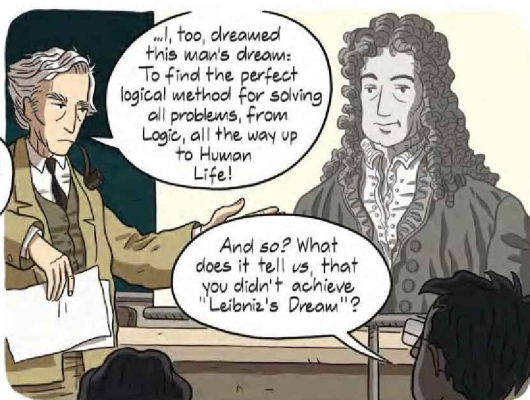
MAIL TO A MARTYR OF THE THIRD REICH!!!

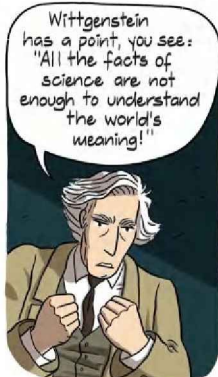
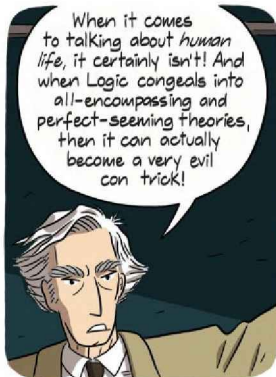
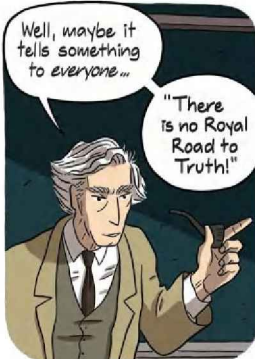
This is a perfect illustration of how the Nazis define "justice", a definition they are doubtless now also busy implementing in Czechoslovakia and, as of three days ago, poor Poland, too. And who knows where next?

And so...

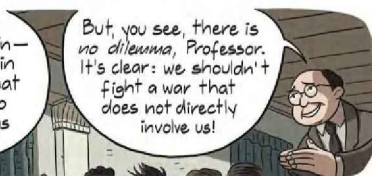
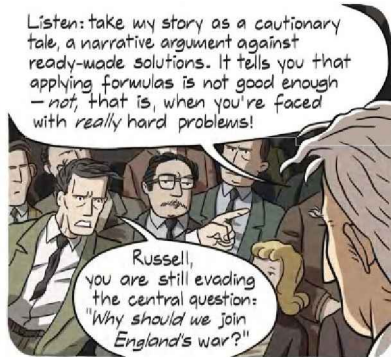
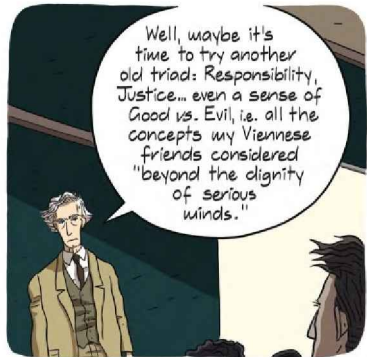
...I finally get to what I consider to be the central question:







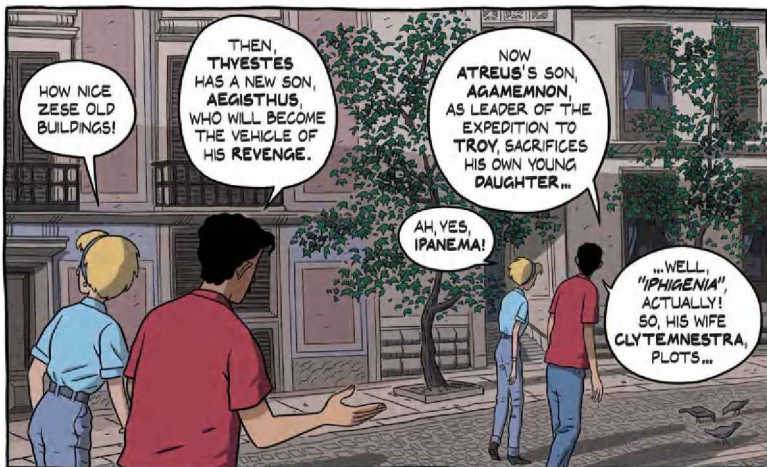






F I N A L E





...WITH AEGISTHUS, NOW HER LOVER, AND TOGETHER THEY KILL AGAMEMNON AS HE RETURNS FROM TROY.

...AND THAT'S PLAY ONE OF THE "ORESTEIA!"



IN PLAY TWO, ORESTES, *i.e.* AGAMEMNON'S SON, IS INSTRUCTED BY THE GOD APOLLO TO KILL HIS MOTHER, FOR PUNISHMENT.

WHERE ARE ALECOS AND CHRISTOS? ZE DRESS REHEARSAL WILL START!



ORESTES IS AWARE OF HIS TRAGIC DILEMMA: TO TAKE OR NOT TO TAKE REVENGE? HE'S AFRAID THAT IN EITHER CASE, SOME GOD WILL BE AFTER HIM!

WHICH IS WHAT HAPPENS!



YOU SEE, HE EVENTUALLY KILLS CLYTEMNESTRA, HIS MOTHER, AND SO THE FURIES, THE OLD GODDESSES OF REVENGE, *a.k.a.* THE "BLOOD-THIRSTY HOUNDS", NOW START TO ASK FOR HIS BLOOD!

ZAT SOUNDS BAD NEWS!



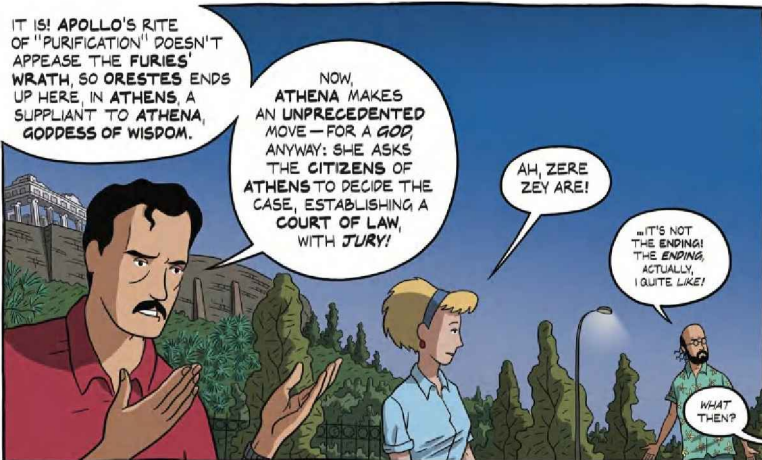
IT IS! APOLLO'S RITE OF "PURIFICATION" DOESN'T APPEASE THE FURIES' WRATH, SO ORESTES ENDS UP HERE, IN ATHENS, A SUPPLIANT TO ATHENA, GODDESS OF WISDOM.

NOW, ATHENA MAKES AN UNPRECEDENTED MOVE — FOR A GOD, ANYWAY: SHE ASKS THE CITIZENS OF ATHENS TO DECIDE THE CASE, ESTABLISHING A COURT OF LAW, WITH JURY!

AH, ZERE ZEY ARE!

...IT'S NOT THE ENDING! THE ENDING, ACTUALLY, IS QUITE LIKE!

WHAT THEN?



IN FACT, I LIKE IT A LOT! ESPECIALLY RUSSELL COMING TO WITTGENSTEIN'S POSITION, THAT ANSWERS TO REALLY IMPORTANT QUESTIONS ARE TO BE CONTEMPLATED "BEYOND WORDS"...

BUT CONTEMPLATING REALITY — NOT MAPS!

WHAT'S UP?

CHRISTOS HAS A PROBLEM WITH THE "FOUNDATIONAL QUEST"!

WRONG! I HAVE TWO PROBLEMS, WITH YOUR VERSION OF IT!

ONE, IT DIDN'T FAIL... AND, TWO, IT WASN'T A TRAGEDY!

IT ISN'T EXACTLY A COMEDY, EH?

GRANTED, THERE ARE TRAGIC PARTS! BUT THE ENDING IS HAPPY, AS IN... IN...

...THE "ORESTEIA"!

"HAPPY" FOR WHOM? CANTOR, GOING INSANE? GÖDEL STARVING HIMSELF TO DEATH OUT OF PARANOIA? HILBERT OR RUSSELL AND THEIR PSYCHOTIC SONS? OR FREGE WITH —

"THE MEANING IS IN THE ENDING!" YOU SAID SO YOURSELF!

SO, FOLLOW THE "QUEST" FOR TEN MORE YEARS...

...AND YOU GET A BRAND-NEW, TRIUMPHANT FINALE...

...WITH THE CREATION OF THE COMPUTER, WHICH IS THE "QUEST"'S REAL HERO!

YOUR PROBLEM IS, SIMPLY, THAT YOU SEE IT AS A STORY OF PEOPLE!

WELL, STORIES DO TEND TO BE "ABOUT PEOPLE"!

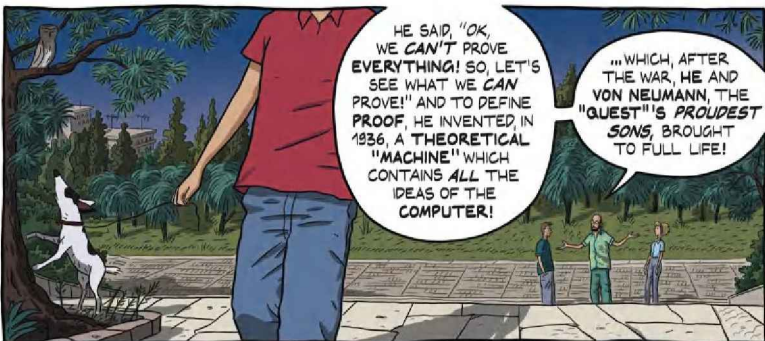
SO, CHOOSE THE **RIGHT** PEOPLE! AND SHOW WHAT THEY **REALLY** DID! ALL WE LEARN OF THE GREAT VON NEUMANN IS HE SAID "IT'S OVER" WHEN HE HEARD GÖDEL!



BUT IT WAS OVER IN A SENSE, WASN'T IT?  
POPP: WENT HILBERT'S "NO IGNORABIMUS"!

BUT THEN CAME THE "QUEST"'S JELINE PREMIER, ITS PARSIFAL...

...ALAN TURING!



HE SAID, "OK, WE CAN'T PROVE EVERYTHING! SO LET'S SEE WHAT WE CAN PROVE!" AND TO DEFINE PROOF, HE INVENTED, IN 1936, A THEORETICAL "MACHINE" WHICH CONTAINS ALL THE IDEAS OF THE COMPUTER!

...WHICH, AFTER THE WAR, HE AND VON NEUMANN, THE "QUEST"'S PROUDEST SONS, BROUGHT TO FULL LIFE!

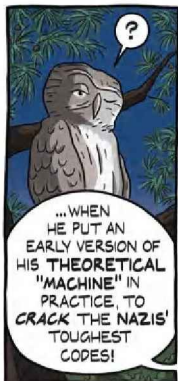
BUT EVEN DURING THE WAR, TURING'S IDEAS BLOOMED...



WOOF!

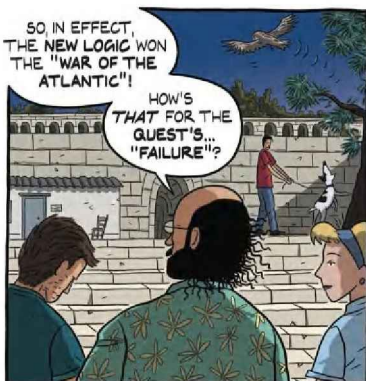
?

...WHEN HE PUT AN EARLY VERSION OF HIS THEORETICAL "MACHINE" IN PRACTICE, TO CRACK THE NAZIS' TOUGHEST CODES!



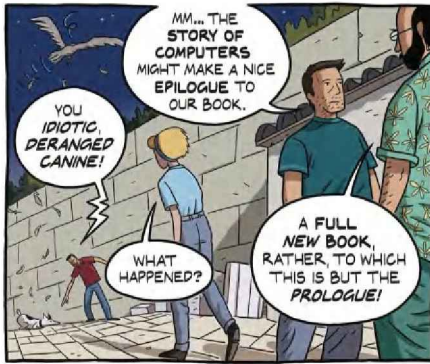
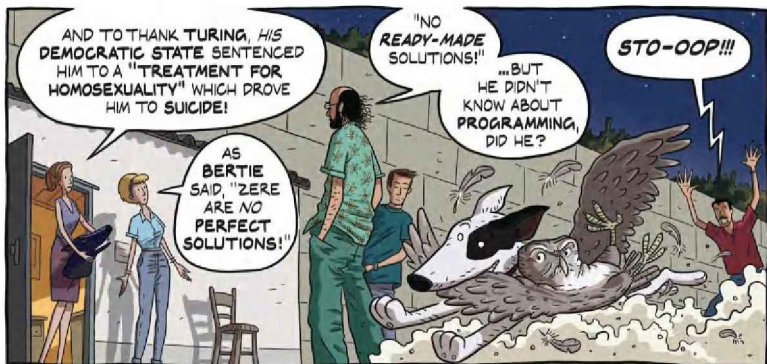
SO IN EFFECT, THE NEW LOGIC WON THE "WAR OF THE ATLANTIC"!

HOW'S THAT FOR THE QUEST'S... "FAILURE"?











NOW,  
FURIES,  
FINISH WITH  
YOUR CASE!

IF YOU ACQUIT THIS MAN, WHO  
KILLED HIS OWN MOTHER...

WE ADDRESS A DREAD  
WARNING TO ALL YOUR CITIZENS,  
DIVINE ATHENA!

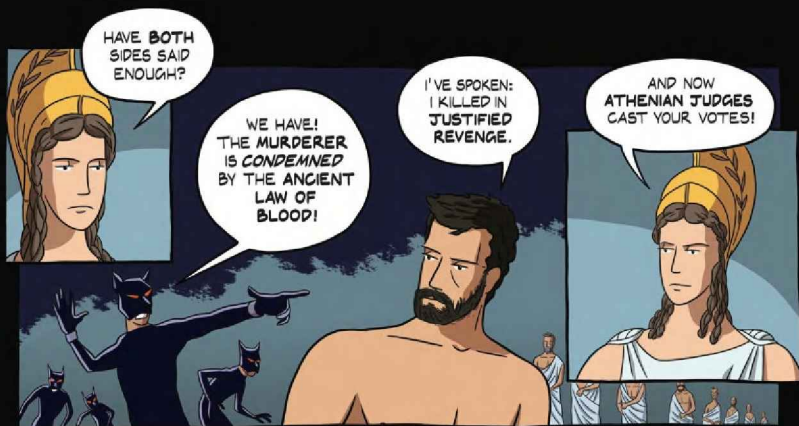
...HER BLOOD  
WILL BE FOREVER  
ON YOUR HEADS!!!

AS IF THE CASE WASN'T  
COMPLICATED ENOUGH, WITHOUT  
THE POOR JURORS ALSO HAVING  
TO TAKE INTO ACCOUNT  
THE FURIES' RAGE!

LIKE THE  
AMERICANS IN  
WORLD WAR TWO:  
IF THEY DECIDE TO  
HELP ENGLAND,  
THEY GET HITLER'S  
RAGE!

NO SUCH THING AS  
AN "EASY PROBLEM"!





HAVE BOTH SIDES SAID ENOUGH?

WE HAVE! THE MURDERER IS CONDEMNED BY THE ANCIENT LAW OF BLOOD!

I'VE SPOKEN: I KILLED IN JUSTIFIED REVENGE.

AND NOW ATHENIAN JUDGES CAST YOUR VOTES!

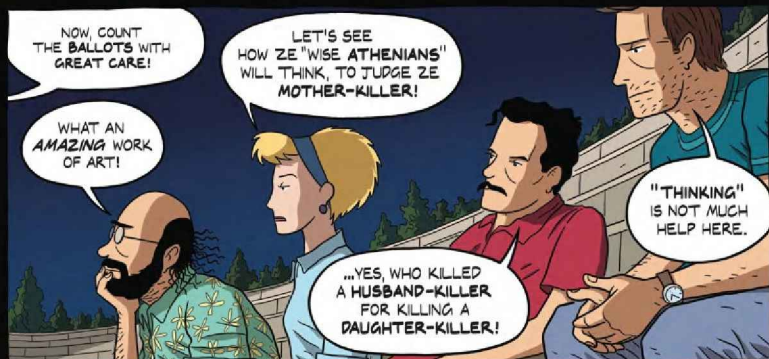


O DARK MOTHER, ARE YOU WATCHING THIS?

EITHER OUR HONOUR THRIVES OR WE ARE FINISHED...

O NIGHT! GUIDE THE JURY TOWARD ANCIENT LAW!

OH GODS... HOW WILL THEY VOTE? THEIR ACT DECIDES WHETHER I LIVE OR DIE!



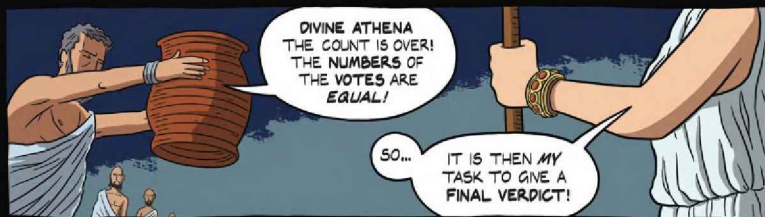
NOW, COUNT THE BALLOTS WITH GREAT CARE!

WHAT AN AMAZING WORK OF ART!

LET'S SEE HOW ZE "WISE ATHENIANS" WILL THINK, TO JUDGE ZE MOTHER-KILLER!

...YES, WHO KILLED A HUSBAND-KILLER FOR KILLING A DAUGHTER-KILLER!

"THINKING" IS NOT MUCH HELP HERE.



DIVINE ATHENA  
THE COUNT IS OVER!  
THE NUMBERS OF  
THE VOTES ARE  
EQUAL!

SO...

IT IS THEN MY  
TASK TO GIVE A  
FINAL VERDICT!



...AND I DECIDE...



...TO GIVE MY VOTE  
TO ORESTES!

HE IS  
ACQUITTED!

O GODDESS,  
YOU'VE  
SAVED ME!

O YOU... YOU...  
YOUNG GODS!

WE'RE  
LOST!!!

OOHHH...

YOU'VE  
RAVAGED  
OUR ANCIENT  
LAWS!

GRRR



AAAARRGGGGHHH!  
I SCREAM, IN  
PAIN!!!

DAUGHTERS  
OF THE NIGHT  
THEY HAVE  
ABUSED US!!!

AARRF... THEY'VE  
DISHONOURD US!

GRRR

ATHENIANS,  
MY HATE-FILLED  
ANGER TURNS  
AGAINST  
YOU!!!

AND ZIS  
ATHENA IS  
GODDESS OF  
WISDOM?

...AND  
ACQUITS ZIS  
MURDERER?





WHAT CAN YOUR CITY GIVE,  
TO MATCH WHAT IT TOOK?  
WHAT CAN REPLACE THE  
POWER YOU USURPED?

IF THERE'S A MAN OR WOMAN  
WHO'S NEVER FELT YOUR ANGER,  
THEY'RE TOTALLY IGNORANT OF  
LIFE'S TRUE NATURE!

SO, STAY,  
ENRICHING  
MY WISDOM WITH  
YOURS! STAY AND  
GUIDE MY CITY  
WITH ME!



SHE IS  
NOT ZE OWL,  
ZE'S ATHENA...  
SHE'S ZE  
FOX!

YOU AND I SHALL BE VENERATED  
SIDE BY SIDE... AND SIDE BY SIDE. WE'LL  
STEER OUR CITIZENS' LIVES  
TOWARDS THE GOOD!

SUCH *POWER*  
I GRANT YOU!!!

OOOHHH...  
HER MAGIC'S  
DOING ITS  
WORK!

I FEEL  
MY RAGE  
DIMINISH,  
SISTER...

WHAT  
SAY YOU,  
SISTER?

I SAY WE  
BETTER TAKE  
HER OFFER!

I SAY  
SO TOO!

IT IS AGREED, THEN,  
DAUGHTER OF ZEUS!  
WE, FURIES, ACCEPT  
YOUR OFFER, SO...

WE'LL LIVE  
WITH YOU,  
*EVERMORE!*

PROBLEM  
PERFECTLY  
SOLVED!

TO  
ACHIEVE  
WISDOM, SHE  
SAYS, YOU  
MUST...

...ALSO  
ALLOW FOR  
A LOT THAT'S  
USUALLY LEFT  
OUT AS  
*UN-WISE!*

COME FORWARD THEN!  
FURIOUS NO MORE, BUT  
*BENEFICENT*, COME  
*BLESS* THE CITY WITH  
YOUR GRACE!



WE, DAUGHTERS OF THE NIGHT, MAKE THIS HOLY PRAYER FOR ALL WHO LIVE HERE...



MAY **MURDEROUS STRIFE NEVER**  
ROAR WITHIN THE CITY, **NOR REVENGE**  
INCITE **BLOOD-THIRSTY WAR!**

MAY THE LAND GIVE **BOUNTEOUS, HAPPY HARVESTS!**  
MAY THE **EARTH BURST FORTH WITH FRUIT,**  
MAKING THE **CITIZENS REJOICE AND CELEBRATE**  
BENEATH A **BRILLIANT SUN...**

...NOT FORGETTING TO PAY  
TRIBUTE TO **HERMES, GOD**  
OF **UNEXPECTED LUCK!**

**REJOICE REJOICE!**  
**REJOICE, YOU HAPPY CITIZENS,**  
WHO LOVE **TRUE WISDOM!!!**







Many thanks for their help to our friends  
Aliko Chapple, Doukas Kapantais, Avraam Kawa,  
Margaret Metzger, Apostolia Papadamaki, Dimitris Sivrikozis,  
Chloe Theodoropoulou, Panagiotis Yiannopoulos

## *Logicomix and reality*

*Logicomix* was inspired by the story of the quest for the foundations of mathematics, whose most intense phase lasted from the last decades of the 19<sup>th</sup> century to the eruption of the Second World War. Yet, despite the fact that its characters are mostly real persons, our book is definitely not – nor does it want to be – a work of history. It is – *and* wants to be – a graphic novel.

Particularly in our reconstruction of Bertrand Russell's life, we've had to wander through an immense amount of material, to select, reduce, simplify, interpret and, very often, invent. Also, though our major characters are based as closely as possible on their real-life counterparts, we have on more than one occasion departed from factual detail, in order to give our narrative greater coherence and depth. Most of these deviations consist in inventing meetings for which there is no historical evidence – or even, in some cases, where there is evidence that they did *not* occur. But these imagined meetings are always based on the actual intellectual interaction of the thinkers involved, conducted in reality either through correspondence or publications.

A few examples of such deviations from fact: from the existing evidence, or lack thereof, it is safe to assume that Russell never met Frege or Cantor in the flesh; there are no indications that he was present in Hilbert's seminal 1900 lecture on the "Problems of mathematics", although he was certainly in Paris a few days earlier, attending the Congress of Philosophy, where he met Peano; there is no evidence whatsoever that he was in the audience during Gödel's "incompleteness" talk – he probably wasn't and Hilbert certainly wasn't, though Von Neumann certainly was, and did say "it's all over" right after. Furthermore, Russell couldn't have visited Frege right after this talk, as the latter had been dead for six years. And although the timing of Frege's rabid anti-Semitic diatribes is incongruous in our book, it is totally true that he wrote them a few years earlier.

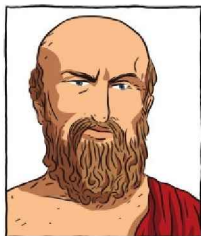
Historically keen readers can have fun locating many more such deviations from fact. For our part, we take comfort in the words of the painter Doménikos Theotokópoulos (better known as "el Greco") explaining the freedoms he took in his painting "Storm over Toledo":

I found it necessary to reduce the size of the hospital of don Juan Tavera, not just because it covered the gate of Bisagra, but also because its dome came up too high, passing the city's skyline. And so, since i've made it smaller and moved it, i think it is better to show its façade, rather than its other sides. As for its actual position in the city, you can see it in the map.

Still, we must add this: apart from the simplification that was necessary to accommodate it into a narrative work of this kind, we have not taken any liberties with the content of the great adventure of ideas which forms our main plot, neither with its central vision, its concepts, nor — even more importantly — with the philosophical, existential and emotional struggles which are inextricably bound with it.

# Notebook

The following notes are by no means necessary for the enjoyment of *Logicomix*, but may give additional information on persons and ideas. A name or term in blue indicates that it also has its own entry, while *italics*, when not used for emphasis, indicate technical terms.



**Aeschylus** One of the three great Greek tragedians, the precursor of Sophocles and Euripides, Aeschylus is the creator of tragedy as we know it. He introduced a second actor into the earlier dramatic form, which only used a protagonist and the chorus, thus also inventing the technique of dramatic dialogue. Born in 525 BCE in Eleusis, near Athens, he fought against the invading armies of Darius at Marathon (490 BCE) and Xerxes at Salamis (480 BCE), the latter battle

also providing the subject matter of his earliest extant play, the *Persians* (first produced in 472 BCE). The titles of seventy-nine of his plays are known to us, but only seven of these have survived in their totality, three of which constitute the *Oresteia* trilogy.



**Algorithm** A methodical, step-by-step procedure described in terms of totally unambiguous instructions, which starts at a specified initial condition and eventually terminates with the desired outcome. Though there is no reason why a well-written cooking recipe, or the instructions for finding a certain geographical location or address cannot be called algorithms, the term originated in mathematics, where it is still mostly used. The word "algorithm" comes from a European transcription of the name of the 9<sup>th</sup> century astronomer and mathematician Al Khwarizmi of Baghdad, who catalogued and championed these methods, having invented many of them. His compendium

of algorithms, the *Hisab al-jabr w' al-muqabala*, is generally considered to be the first algebraic treatise, the very words *al-jabr* in it also providing the root for our word "algebra". An example of a simple mathematical algorithm is the method we learn in elementary school for adding two integers: "write the two numbers one under the other with their rightmost digits justified to the right; add their last digits; if the sum is less than 10, write that number right under the other two; if it is greater than 10, write the second digit of the sum right under the other two, and add the first digit to the sum of the digits immediately

to the left..." and so on. Probably the earliest sophisticated Western algorithm is the one given in Euclid's *Elements* for computing the greatest common divisor of two non-negative numbers. Algorithms gained prominence in the West in the 15<sup>th</sup> century with the introduction of the decimal system, which, in stark contrast with the roman numerical system, was amenable to fast calculations, such as the one described above. Numerical algorithms played a central part in the scientific and technological revolutions. Today, algorithms are usually coded in advanced notations called *programming languages*. They are often transmitted over the internet, and constitute the *software* that is the workhorse, platform, and backbone of computers and the internet.



**Aristotle** Born in 384 BCE, in Stageira, Chalcidice, Aristotle is, with Plato, the most influential of greek philosophers. After he left Plato's Academy, Aristotle developed his own philosophy, which departed from his teacher's in its emphasis on the systematic observation of reality and the attempt to shape general, inductive laws. Perhaps his most lasting contribution is the systematization and exposition of logic in a series of works which later commentators edited collectively

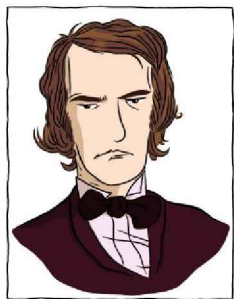
as the *Organon* ("instrument" or "tool"). The books comprising the *Organon*, i.e. *The Categories*, *On Interpretation*, *The Prior Analytics*, *The Posterior Analytics*, *The Topics* and the *Sophistical Refutations* formed the core of the canon of the study of logic until the 19<sup>th</sup> century. At the heart of Aristotle's logic is the combination of non-ambiguous statements in *sylogisms* to create new statements, different from the original but following necessarily from them. Aristotle also had a huge and lasting influence on mathematics, mainly through his emphasis on the notion of *first principles* from which any logical investigation must begin. it was this notion that found its mathematical incarnation in Euclid's concept of the axioms from which every theory has to begin. Aristotle died in 322 BCE.



**Athena** The ancient greek goddess of wisdom, as well as of the arts and the city. Athena sprang in full armour from the head of Zeus, father of the gods, whose favourite child she became. Athena was the patron goddess of ancient Athens and greatly beloved of the Athenians, to whom, according to legend, she gave the gift of the olive tree. The Parthenon, in the centre of the Acropolis,

is a temple to her – the word comes from *parthenos*, meaning “virgin”. Athena’s role in Aeschylus’ trilogy, the *Oresteia*, gives her a central role in the origin myth for the Athenian democratic invention of trial by jury, a system based on reason, as opposed to the older ones, where juridical authority emanated from a ruler’s absolute power.

**Axiom** Since the time of Euclid, who was working in the wake of Aristotle’s philosophy of logic, mathematicians agree that a workable theory must rest on some (few) agreed-upon *first principles* that don’t require proof. This is a logical necessity if one wants to avoid, on the one hand, *infinite regression* (endlessly having to base something on something else) and, on the other, *circuitous* thinking (constructing proofs for statements which, however indirectly, assume the original statement to be true in the first place). Up to the 19<sup>th</sup> century, axioms were generally considered to be *self-evident* truths about the world, a view more or less still valid in Frege’s idea of axioms as the reflection of an ulterior logical reality. After Hilbert, however, and under the influence of the mathematico-philosophical school of *formalism*, which developed from his ideas, axioms came to be seen as existing independently of any outside reality, the only requirements of an axiomatic system being: for the individual axioms their *grammatical correctness* (in other words, their being well-formed according to the rules of the logical language in which they are expressed), and *independence* (their not being derivable from the other axioms of the particular theory); and, for the whole set of axioms, its internal *consistency* (not containing axioms which contradict one another).



**Boole, George** Born in 1815, Boole was a largely self-taught mathematician who later became a professor of mathematics and logic at Queen’s College in Cork, Ireland. His great contribution to mathematics is in the field of logic. In his book *An Investigation of the Laws of Thought*, Boole developed the idea that logical propositions can be expressed in a purely symbolic language which allows them to be manipulated by operations, similar to the operations of elementary arithmetic. At the heart of Boole’s work is the idea

of a *propositional calculus*, constructed somewhat as Leibniz imagined it. The “Boolean search” on the internet, involving use of the logical *connectives* “and”,

"or" and "not", can be traced directly back to his ideas. Yet, despite the great value of his work in mathematizing logical arguments, Boole did not offer any great insights into the study of logic itself, having worked wholly within Aristotle's classical model. In Boole's system, *symbols* such as  $x$  and  $Y$  (essentially they are variables that can take only the two values 0 and 1) are joined via the three connectives mentioned above, as well as the "implies" connective envisaged by Aristotle. (Interestingly, the Stoic Chrysippus had already identified these connectives in the 3rd century BCE.) The application of algebraic identities, such as the three below, allow a logician to simplify logical expressions and deduce useful conclusions from them:

$$(x \text{ or } Y) = (Y \text{ or } x)$$

$$\text{not}(\text{not } x) = x$$

$$\text{not}(x \text{ and } Y) = (\text{not } x) \text{ or } (\text{not } Y)$$

What this logical formalism is lacking is the ability to express semantic connections between propositions. So, for example, there is no way to denote in the above that  $x$  and  $Y$  may stand for the two propositions "Plato is older than Socrates" and "Socrates is older than Plato."

This weakness is remedied in the predicate calculus. Boole died in 1864.

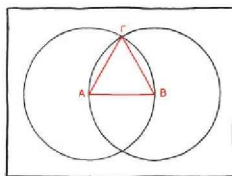


**Cantor, Georg** Born in 1845, Cantor studied under some of the greatest mathematicians of his time, including Richard Dedekind and Karl Weierstrass. He spent the greatest part of his career teaching at the University of Halle, where he wrote his seminal papers demonstrating the great power of the ideas of set theory. His most famous theorem is that the set of so-called *real* numbers (all the numbers on the *number line*, i.e. the *natural* numbers 1, 2, 3... etc., together with the decimals, including 0 and the negatives) is *uncountable*, in other words cannot be put into a one-to-one

correspondence with the whole numbers 1, 2, 3, ... etc. On the contrary, as Cantor had already proved, the set of all *rational numbers*, i.e. all fractions of natural numbers, such as  $\frac{2}{3}$  or  $\frac{11}{476}$ , is *countable* and can be put in such a correspondence. As both countable and non-countable sets have an infinity of elements, Cantor's results essentially proved that there are various, mutually exclusive kinds of infinity. As his theorems were extremely counter-intuitive and thus totally unexpected, they created much skepticism about set theory in the mathematical community.



One of Cantor's teachers, the great mathematician Leopold Kronecker, as well as the mathematical giant Henri Poincaré were strongly critical of sets, though the other mathematical giant of that time, David Hilbert, was one of Cantor's greatest supporters. The identification of two distinct 'sizes' of infinity in the set of real numbers, a smaller and a larger one, ushered in the question of whether there could exist a third kind: could there be a subset of the real numbers that is neither countable nor can be put in one-to-one correspondence with the reals? Cantor conjectured that none exists, a guess ever since called "the Continuum hypothesis" – the *Continuum* being another name for the number line. Cantor worked towards a proof of the Continuum hypothesis for many years, but never achieved it. In 1940, Kurt Gödel proved that the Continuum hypothesis is *consistent* with the prevailing axiomatic system of set theory (which does *not* amount to a proof of it). In 1963, the young American mathematician Paul Cohen proved that it is *independent* of it, i.e. that no real proof of the hypothesis can be established from it, or, alternatively, that the axioms of set theory are consistent with the hypothesis being either true *or* false. This discovery earned Cohen a Fields medal, a distinction often called "the Nobel Prize of mathematics". Cantor suffered from severe emotional problems and was repeatedly hospitalized with a diagnosis of melancholia, which certain historians of mathematics have ascribed to the hostile reactions of some mathematicians to set theory, and others to the constant anxiety resulting from his fruitless attempt to prove the Continuum hypothesis. In the last decades of his life Cantor did no mathematical work, but wrote extensively trying to substantiate two strange theories: a) that the plays of Shakespeare were in fact written by the Elizabethan philosopher Sir Francis Bacon, and b) that Christ was the natural son of Joseph of Arimathea. The second of these is a basic component of many variations of the Holy Grail legend, and a standard part of esoteric lore. Cantor died in a mental asylum, where he had been interned against his will, in 1918.



**Euclid** Born around 325 BCE, Euclid is the earliest Greek mathematician whose work is extant in the form in which he actually gave it – theorems of earlier mathematicians survive only as transcribed by others. He lived and worked in Alexandria, where he was associated with the great Library. His opus magnum, the *Elements*, has been a best-seller for twenty-three centuries, and is the book with the most editions in the Western world, after the Bible. Though many of the theorems appearing in it are probably not Euclid's own discoveries, the work of compilation, classification

and presentation of the existing mathematics of his day is totally his own. The *Elements* is a majestic conceptual edifice which, inspired by Aristotle and his work on logic, starts from definitions and first principles, the axioms (*aitēmata* – literally “requests” – in euclid’s original greek) and then proceeds to arrive at all the theorems through rigorous proof. Though later students of logic, especially at the time of the quest for the foundations of mathematics and after, have criticized euclid for relying too much on geometric insight or taking many more things for granted than his axioms, the influence of the *Elements* has been colossal, and it is rightly considered to be the fountainhead of the mathematical method. euclid died around 265 BCE.



**Foundations of Mathematics** Since the time of Pythagoras, mathematicians have wondered about the nature of mathematical truth, the ontology of mathematical entities and the reasons for the validity of proof and, more generally, mathematical knowledge. From the enlightenment until the middle of the 19<sup>th</sup> century, the prevailing scientific ideology saw mathematics as the only way of reaching a truth that is final, absolute and totally independent of the human mind’s capacity

to understand it. The basic notions of mathematics were thought to reflect essential properties of the cosmos and the theorems to be the truths of a higher reality. This absolute faith in mathematics is reflected in the crowning of the discipline as the “Queen of the Sciences”, a title whose previous holder, significantly, was theology. This view is usually termed *mathematical Platonism*, having its roots in the views of Plato – and, at least partly, Pythagoras before him – on the transcendent ideas (*eidē*). Yet, in the 19<sup>th</sup> century this traditional belief was undermined in the minds of some people and eventually led to a serious *foundational crisis* in mathematics. The first of the discoveries which caused this loss of faith, dating from the time of the renaissance, was that of the *imaginary numbers* (i.e. those involving the square root of minus one). But in the 19<sup>th</sup> century the appearance of *non-Euclidean geometries* strengthened the arguments against the “self-evident” truth of the axioms. The most troublesome of all mathematical concepts, though, was that of infinity. Problems concerning the mathematical handling of the infinite had first been alluded to by Zeno, in his paradoxes, resurfaced with the invention of the calculus in the 18<sup>th</sup> century and the counterintuitive and ill-defined concept of an *infinitesimal*, and peaked in the last two decades of the 19<sup>th</sup> century, most especially with set theory and Georg Cantor’s

results on infinite sets. The problems that came to the surface via set theory — chief among them Russell's Paradox — culminated in severe doubts about "self-evident" truths and thus, indirectly, about the value of all mathematical knowledge. It was principally the wish to overcome these doubts that fuelled the quest for secure foundations. The "Program" proclaimed by David Hilbert in the early 1920's bearing his name, expresses the most optimistic version of the foundational dream: the creation of a formal system for all mathematics, also containing a proof that this axiomatization is *consistent* (i.e. can lead to no contradictions), *complete* (leaves no unprovable truths) and *decidable* (one is able to decide in every occasion whether a formula follows from the axioms or not, through the application of a set of algorithms.)



**Frege, Gottlob** Born in 1848, Frege spent the greatest part of his mathematical life as a professor at the University of Jena. He is generally considered to be the father of modern logic, whose notation and method he expounded first in his *Begriffsschrift* (which literally translates from the German as "concept script"), published in 1879. In it, Frege departed from the earlier logicians working in the wake of Aristotle, by explicitly introducing the notion of *variable* in logical statements. In the place of the older type of statements like "Socrates is a man", he introduced propositions like "x is a man", propositions that can

be true or false according to the value given to x — this particular one, for example, is true if x is equal to "Alecoc" but false if it's "Manga". Frege also invented the notion of *quantifiers*, the *universal* (written  $\forall$ ) which makes a statement true "for every x"; and the *existential* (written  $\exists$ ) which says that "there exists an x" which makes a statement true. He later applied his new logical system to the quest for the foundations of mathematics. His *Grundgesetze der Arithmetik* (*The Basic Laws of Arithmetic*) is the first great work of the school of *logicism*, whose central tenet is that mathematics is essentially a branch of logic. The first volume of the *Grundgesetze* was published in 1893 and the second, containing the addendum on Russell's Paradox, in 1903. Though Frege's logical symbolism has been abandoned as particularly cumbersome, most of the basic concepts and methods he invented still form the backbone of logic. After the *Grundgesetze*, Frege didn't do any important foundational work. In the last decades of his life he became increasingly paranoid, writing

a series of rabid treatises attacking parliamentary democracy, labour unions, foreigners and, especially, the Jews, even suggesting "final solutions" to the "Jewish problem". he died in 1925.



**Gödel, Kurt** He was born in 1906 in the town of Brünn, moravia, then a part of the Austro-hungarian empire (the city now called Brno, in the Czech republic). Gödel studied mathematics in Vienna, where he became fascinated with mathematical logic and the question of the foundations of mathematics. in his doctoral dissertation, he advanced Hilbert's Program by proving his *Completeness Theorem*, a result establishing that all valid statements in Frege's *first-order logic* can be proved from a set of simple axioms. in 1931, however, he proved the *Incompleteness Theorem for second-order*

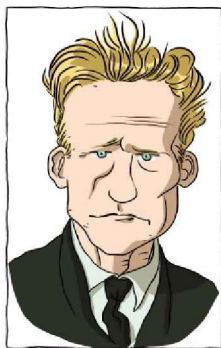
*logic*, i.e. for a logic powerful enough to support arithmetic and equally or more complex mathematical theories. Gödel became one of the youngest members of the Vienna Circle, though his deeply-ingrained, idealist belief in the independent, Platonic existence of mathematical reality eventually alienated him from the other members, who embraced a materialist-empirical worldview. during the late thirties, Gödel was hospitalized twice for severe melancholia. in 1940, after the *Anschluss*, i.e. the annexation of Austria to Nazi germany, he managed to escape the country with his wife and took the trans-Siberian route to the United States. he became one of the first members of the institute for Advanced Study at Princeton, where he spent the rest of his life. his most important mathematical result from this period is the proof that Cantor's Continuum hypothesis is *consistent* with the axioms of set theory (i.e. that it would not be in contradiction with them, if true). At Princeton, Gödel developed a close friendship with Albert einstein and worked for a while on the theory of relativity, establishing the mathematical possibility of a non-expanding, rotating universe, in which time travel can be a physical reality. in later life, Gödel became increasingly paranoid. he died in January 1978, at the Princeton hospital, where he had been admitted for the treatment of a non-life-threatening urinary tract problem. The cause of his death was malnutrition: he refused to eat for fear that the hospital staff was attempting to poison him.



**Hilbert, David** Hilbert was born in 1862 in Königsberg, Prussia (now Kaliningrad, Russia) and spent the greatest part of his life at the University of Göttingen, the world's most renowned mathematical centre at that time. He is one of the greatest mathematicians in history and, with Henri Poincaré, the greatest of his era. He made important contributions to many branches of mathematics including

invariant theory, algebraic number theory, functional analysis, the calculus of variations, the theory of differential equations and more, also pioneering new methods of proof. In 1899 he published *Grundlagen der Geometrie* (*Foundations of Geometry*), a book which gave geometry a firm basis, with new axioms, therein improving on the work of Euclid. In his famous 1900 talk at the international Congress of mathematicians, in Paris, he attempted to give a bird's-eye view of the mathematics of the twentieth century, by way of twenty-three great open questions. Of these, now renowned "hilbert's problems", eleven have been fully solved, seven partly, while the rest – the eighth, also known as "the riemann hypothesis" is the most famous of these – are still unsolved. The Second Problem is the one demanding a proof of the *consistency* (the *completeness* was considered more or less obvious) of arithmetic – and it was this that spurred on a lot of the work on the foundations and logical structure of arithmetic, including Gödel's. In the 1920s, his ideas of the previous decades related to the foundations of mathematics culminated in what became known as "hilbert's Program", i.e. a project to formalize all mathematics on an axiomatic basis, including a proof that this axiomatization is consistent. Hilbert's battle cries of "in mathematics there is no *ignorabimus*" (i.e. no "we shall not know") and "we must know, we shall know" – the latter spoken only a few days before Gödel's first announcement of his Incompleteness Theorem – encapsulate the quintessence of foundational optimism. Though the results of Gödel, Alan Turing and Alonzo Church put an end to hilbert's grand ambition, the Program continued to exert a great influence on logic and foundational matters, and especially the development of proof theory. Though in outward appearance and behaviour hilbert gave the impression of a paragon of normality and mental health, the way he treated his only son, Franz, raises questions. When the boy was diagnosed with schizophrenia, at age 15, his father sent him off to an asylum, where he spent the rest of his life. Hilbert never visited his son. He died in 1943.

**Incompleteness Theorem** In 1931, the 25 year-old Kurt Gödel proved two theorems that are sometimes referred to as "the" incompleteness Theorem – though occasionally this form is used to denote the first of these. The *completeness* of a logical system is the property that every *well-formed* (i.e. grammatically correct by the rules of the system) proposition in it can be proved or disproved from the system's axioms. Gödel's earlier *Completeness Theorem* shows that there is a simple such axiomatic system for *first-order logic*. However, the holy grail of Hilbert's Program was to create a *complete* and *consistent* axiomatic system that can support *arithmetic*, i.e. the mathematics of whole numbers. Such a system would require *second-order logic*, i.e. a system that is also able to accept sets as values of variables. Gödel shocked the mathematical world by proving, in his famous paper "on Undecidable Propositions in the *Principia Mathematica* and related Systems", that any consistent axiomatic system for arithmetic, in the form developed in the *Principia*, must of necessity be *incomplete*. More precisely, the first of the two incompleteness Theorems establishes that in a logical axiomatic system rich enough to describe properties of the whole numbers and ordinary arithmetic operations, there will always be propositions that are grammatically correct by the rules of the system, and moreover *true*, but cannot be proven within the system. The second incompleteness Theorem states that if such a system were to prove its own consistency it would be inconsistent. This was a new, devastating blow to Hilbert's Program, with its goal that a strong axiomatic system should be equipped with a proof of its own consistency.



**Intuitionism** This is the philosophy of mathematics created by the great Dutch mathematician Luitzen Egbertus Jan Brouwer (1881-1966), though some consider Henri Poincaré, with his strong belief in the role of intuition in mathematics, a clear precursor. Intuitionism is based on the belief that intuition and time are fundamental to mathematics, which cannot be made a-temporal or *formal* in the sense of Hilbert. Contrary to what *logicians* like Frege and Russell thought, Brouwer was convinced that logic is founded upon mathematics rather than the other way round. Also, he was totally

against the theorems of Georg Cantor in the theory of sets, considering

them ill-formed. Time-hallowed logical laws, such as that of the *excluded middle*, and mathematical techniques in use since the time of the ancient greeks, such as the *reductio ad absurdum*, were put on trial and their use condemned. In fact, Brouwer believed that all the theorems making use of these in their proofs – where infinite sets of mathematical objects were concerned – should be excised from the body of mathematics, a view that made the brilliant British logician and mathematician Frank ramsey call intuitionism “mathematical Bolshevism”. Although his logic and mathematics were formalized by his student Arend heyting, Brouwer remained skeptical towards any such attempt to the end of his life.



**Leibniz, Gottfried** This great german philosopher, mathematician, scientist and student of logic was born in 1646. He served in the courts of several german rulers as diplomat, political advisor and historian, all the while pursuing his theoretical studies. He invented the *infinitesimal calculus* concurrently with, but independently from, isaac Newton,

also proposing the notation for its operations that is still in use today.

He was a strong proponent of *philosophical optimism*, with his theory that our world is the “best of all possible worlds”, having been created by a god who is both loving and almighty. He is considered the most important logician after Aristotle and before Boole, having envisioned the *calculus ratiocinator*. This was a kind of computational propositional logic that would enable completely rigorous and rational decision-making which could eliminate all disagreement among rational (as Leibniz thought them) human beings. Sadly, Leibniz did not manage to realize this most coveted of his many projects. He died in 1716.

**Logic** The term covers a broad spectrum of disciplines – not unexpectedly, as it derives from one of the semantically richest greek words, *logos*, some of whose meanings are *word, speech, thought, reason, ratio, rationality*, and/or *concept* – but can perhaps be best described as the study of methodical thinking, deduction and demonstration. The books of Aristotle’s *Organon* present an extensive study of the deductive patterns called *sylogisms*, which for over two millennia were considered practically synonymous with logical thinking. Until the middle of the 19th century, logic was considered a branch of philosophy. But with the advent of Boole and his

algebra of propositions and, more importantly, Frege and his "concept script" which led to a predicate calculus, it increasingly came within the province of mathematics. The new logic revealed both the underlying mathematical nature of the subject and its potential role in the creation of solid foundations of mathematics. The basic claim of the school in the philosophy of mathematics known as *logicism* – the school founded by Frege, of which Bertrand Russell was one of the primary exponents – was that all of mathematics can be reduced to logic or, in other words, that mathematics is essentially a *branch* of logic. After the years of the foundational quest, however, and especially after Gödel's results, logic became a well-developed, diversified field in the interface between philosophy and mathematics. In the second half of the 20<sup>th</sup> century it also found unexpected applications in computer science, where it provides solid foundations for the design and verification of software and hardware, as well as for databases and artificial intelligence.



***Oresteia*** Written by Aeschylus and first performed in the theatre of Dionysus, in Athens, two years before the poet's death, in 458 BCE, it is the only extant trilogy of Greek dramas – although the satirical play *Proteus*, intended to be performed after the trilogy, is missing. In the trilogy's first play, the *Agamemnon*, the eponymous hero and leader of the Greek forces in Troy returns a victor to his hometown

of Argos, with the captive prophetess, Cassandra. Though his wife, Clytemnestra, at first appears to rejoice at his return, she has other plans. She and her lover, Agamemnon's cousin Aegisthus, murder Agamemnon and become the new sovereigns of Argos. In the *Libation Bearers*, the second play, the chorus of women accompanies Agamemnon's daughter Electra to her father's tomb. The forlorn Electra is hoping for revenge, which she can only carry out with the help of her brother, Orestes, who is in exile. When Orestes clandestinely returns to Argos, he and Electra plan and execute the murder of Aegisthus and then, in a highly dramatic scene in which Clytemnestra bares her breasts before his naked sword, Orestes also kills her, his own mother. The third play, the *Eumenides*, or "beneficent ones", is one of the most unusual in the history of drama: all its speaking parts, apart from that of Orestes himself, are taken up by gods or other supernatural entities. The chorus consists of the *Erinyes* or Furies, archaic goddesses of revenge, who chase Orestes from the temple at Delphi, where



he has been ritually purified by the god Apollo, to Athens.

In a totally unprecedented move – for a god anyway – Athena, the patron god of Athens, decides to let the citizens of Athens judge orestes' case, thus giving a mythological origin-story for the democratic innovation of a court of law, with citizen jury. The trial and its aftermath develop as shown in our book's finale, though our text is only an approximate translation, slightly adapted, of Aeschylus' original words.



**Peano, Giuseppe** Born in 1858, this great Italian mathematician and logician spent the greatest part of his creative life as a professor at the University of Turin. Though his ideas were not as influential as Frege's in the search for the foundations of mathematics, Peano, like Frege, created a notation for *first-order logic* and a system of axioms for arithmetic, that is still in use – in fact, our arithmetic is formally called *Peano arithmetic*. He influenced Bertrand Russell

greatly, especially with his logical notation, which was much more user-friendly than Frege's. Peano believed that all mathematics could be formalized and expressed in a common, minimal language that originates from just a few axioms. But when he tried to present his own version of this universal mathematics in textbook form and use it for teaching, his students revolted, eventually causing the book's withdrawal. Inspired by his attempts to unify all mathematics by use of a common logical language, Peano later created an international auxiliary natural language, for use among people of different linguistic backgrounds, based on a simplified form of Latin which he called *Latino sine flexione*. However, like so many other artificial international languages, such as *Esperanto*, *Volapük*, *Ido* – all of them the offspring of an overoptimistic age – Peano's brainchild proved to be a mere pipe dream. Peano died in 1932.



**Poincaré, Henri** Born in 1854 in Nancy, France. Although he studied engineering at the école Polytechnique and the école de mines, Poincaré was to become, with David Hilbert, the greatest mathematician of his time. He has been called the "last universal mathematician", i.e. the last one to have profound knowledge of all the mathematics

of his time. He made important contributions to many diverse fields of mathematics, among them differential equations, automorphic functions, the theory of several complex variables, probability and statistics. With his *Analysis situs* he essentially created the major 20<sup>th</sup> century field of *algebraic topology*, and his work on the 3-body problem laid the groundwork for what is now called *chaos theory*. Despite his many great innovations, Poincaré was an extremely practical man, involved to the end of his life – alongside of his mathematical research – with the most down-to-earth of affairs, as for example the inspection of mines and an engineering project to make the Eiffel tower function as a huge antenna broadcasting time signals to navigators. He was probably the last of the great mathematicians to adhere to an older conception of mathematics, which championed a romantic faith in intuition over rigour and formalism. This stance was made famous by his reaction to the set theory of Georg Cantor as a “disease, from which mathematics will eventually be cured.” His views on mathematical creation, encapsulated in his saying that “logic is barren, unless fertilized by intuition,” are seen by many as the precursor of Luitzen Brouwer’s school of intuitionism, a theory at the antipodes of Hilbert’s strict *formalism*. Poincaré died in 1912.

**Predicate calculus** Often used synonymously with *predicate logic* and *first-order logic*, the predicate calculus is Frege’s extension of the propositional logic developed by Boole. In the predicate calculus, elementary propositions (or *predicates*) are composite objects of the form  $P(a, b, c, \dots)$ , where  $P$  is a symbol in the language, and  $a, b, c$ , etc. are *constants* or *variables*. For example, if “older” is a propositional symbol, “Plato” is a constant and “ $x$ ” is a variable, then “older(Plato,  $x$ )” is a well-formed proposition, describing that Plato is older than  $x$ . Propositions of this type can then be combined by Boole’s connectives “and”, “or”, “not” and “implies” and prefixed by Frege’s quantifiers, such as “for all  $x$ ” (written  $\forall$ ) and “there exists  $y$ ” (written  $\exists$ ). Thus, “there exists  $x$  older than Plato” means that there is (at least) one individual who is older than Plato. Evidently, this is a much more ambitious attempt at creating Leibniz’s *calculus ratiocinator* than Boole’s simpler formal logic. By employing symbols from various fields of mathematics (such as “ $<$ ”, “ $+$ ”, and so on) one can create predicates expressing mathematical statements in this formal, logically exact language. For example, the theorem in arithmetic stating that every integer is either odd or even can be written thus:

$$\forall x \exists y (x=y+y \text{ or } x=y+y+1)$$

Rigorously defined, the version of the predicate calculus called *first-order logic* employs simple mathematical objects as variables, whereas in *second-order logic* variables can also be sets, making possible statements like "there is a set S". This, more powerful language, can express all known mathematics. Whether a sentence in the predicate calculus, first- or second-order, is true or false depends on the *model* whereby the sentence is interpreted. Thus, for example, the simple arithmetical theorem given above is true of the whole numbers in the ordinary interpretation of "+", but becomes false if we interpret the symbol "+" as multiplication. However, some sentences – called *valid* – are true independently of interpretation, because they embody basic properties of Boolean connectives and quantifiers. Kurt Gödel's *Completeness Theorem* provides a simple, complete axiomatic system for proving validity in *first-order logic*.

***Principia Mathematica*** The extremely influential, but highly controversial, essentially unfinished work in which Alfred North Whitehead and Bertrand Russell attempted to rescue Frege's grand project to create foundations of mathematics built on logic, in the wake of the crisis brought on by Russell's Paradox. The title *Principia Mathematica* (i.e. "Principles of Mathematics") in itself provoked controversy, as it is the exact same as that of Newton's greatest work; many in the British mathematical community thought this choice to be in bad taste, if not actually blasphemous. The three volumes of the *Principia*, published in 1910, 1912 and 1913, were based on a developed version of Russell's *theory of types*, the so-called "ramified", which imposed a hierarchical structure on the objects of set theory. This could not be made to yield the required results, however, without the addition of what Russell called an *axiom of reducibility*, which eventually became one of the main reasons for negative criticism of the whole work. Logicians found this axiom extremely counter-intuitive, a far-fetched and basically artificial method to sweep the very problem it was trying to solve under the rug. Despite the fact that the *Principia* fell short of its authors' immense ambition, it had a huge influence on the shaping of modern logic, its greatest effect possibly being the inspiration and context it provided Kurt Gödel for his groundbreaking discovery, the Incompleteness Theorem.



**Proof** The process of arriving at the logical verification of a mathematical or logical statement, starting from a set of agreed-upon first principles (these could be either axioms or already proven statements, deriving from these axioms), and proceeding by totally unambiguous and unabridged logical steps or *rules of inference*. The demonstrations of geometric propositions in Euclid's *Elements* were considered for over two millennia to set the standard of excellence to which mathematical proof should aspire. Yet, towards the end of the 19th century his method came under logical and philosophical scrutiny and was found to lack, principally, in two directions: a) in its sense of the logical "obviousness" of the axioms, and b) in its logical gaps, where intuition – which, in euclid's case was mostly visual-geometric – took over from strict application

of a formal system of rules. In a sense, Frege's and Russell and Whitehead's *logician* project was developed as a reaction to the imperfections found in euclid's proofs, as well as all those developed in his wake. The logicians, as well as the *formalists* working on the foundations of mathematics, aimed at a fully developed theory and practice of rigorous proof, by which arithmetic (as the basis of all mathematics) would begin from a small number of consistent axioms, and eventually lead, via proof, to the full range of truth. Hilbert's seminal question, which he called the *Entscheidungsproblem* ("decision problem"), posed in 1928 and answered seven years later by Alan Turing, is equivalent to the demand for a totally powerful apparatus of proof, which can provide a *provable* or *unprovable* response to any mathematical statement by virtue of a rigorous algorithm.



**Russell, Bertrand** Born in Wales, in 1872, Bertrand Arthur William, the Third earl russell – this is his full name, by virtue of his noble descent – was the grandson of the important politician Lord John russell, whose title he eventually inherited. An orphan at the age of four, russell was raised by his paternal grandparents, and after his grandfather's death two years later, exclusively by his grandmother, Lady russell. He grew up at the family home of

Pembroke Lodge, in richmond Park, to the west of London. Russell is now perhaps best known to a wider public for his work in philosophical exposition. His *History of Western Philosophy*, published in 1945, remains to this day a classic of idiosyncratic, yet intelligent and highly readable exposition of complex ideas. And while his later work as a pro-peace and anti-nuclear activist also earned him international fame, russell's greatest contribution is in mathematical logic, ranking him, along with Aristotle, Boole, Frege and Gödel, with history's greatest logicians. Despite the momentous importance of his work in the establishment of a scientific logic, its direct influence on Gödel's great discoveries, and the indirect on the Vienna Circle's "scientific worldview" and the philosophies of *logical positivism* and *logical empiricism*, russell's work in logic essentially ends with the *Principia Mathematica*, the book he co-authored with Alfred North Whitehead, completed just before he turned forty. Russell considered the *Principia* essentially a failure, as it fell short of his –and the other logicians'– grand ambition, of founding mathematics securely on logic. Russell married four times and fathered three children. His first son, John, as well as John's daughter, were diagnosed as schizophrenics, and the latter committed suicide. This pathology was very possibly another instance of the streak of mental illness running in the family, manifest both in russell's uncle William and his aunt Agatha. During the last decades of his life, russell gave all his energy to the struggle for nuclear disarmament, becoming an emblematic figure of pacifism. He died in 1970.



**Russell's Paradox** discovered in 1901, as Russell was working on his first book on the foundations of mathematics, the *Principles of Mathematics* (published in 1903), the Paradox, in the form originally expressed, shows an essential flaw in Cantor's set theory, developed from Bolzano's simple concept of a "collection of elements with a common property". By the generality of this definition, which Frege extended to the realm of logic, one can speak

of a "sets of sets" and thus, eventually of the "set of all sets". Of the elements of this all-encompassing set one defines the property of "self-inclusiveness", i.e. of a set containing itself as an element. Thus, for example, the set of all sets is a set (and thus contained in itself), as is the set of all entries in a list (it can appear as an entry in a list),

but the set of all numbers is not a number and thus not contained in itself. By virtue of this property, we can define the "set of all sets which don't contain themselves", and ask, with the young russell, the question: "does this set contain itself or not?" See what happens: if it *does* contain itself, it follows that it is one of the sets which don't contain themselves (as this is the property that characterizes elements of this set) and thus cannot contain itself. But if it *doesn't* contain itself, then it does not have the property of not containing itself, and thus does contain itself. This situation, in which assuming something implies its negation, and vice versa, is called a *paradox*. When a paradox, such as russell's, arises in a theory, it is a sign that one of its basic premises, definitions or axioms is faulty. Though historically developed within the context of the theory of sets, russell himself later viewed his paradox as essentially having to do with self-reference, i.e. with statements referring to themselves, such as euboulides' "i am now lying to you."



**Self-reference** Literally, the quality of a statement of referring to itself. However, it is also used more generally in logic to characterize statements which include themselves within their scope of reference, as in the "barber" story used to explain Russell's Paradox. The barber lives in a town wherein a law decrees that

"all residents of the town must either shave themselves or be shaved by the barber." This law is *self-referential* as the barber, apart from being "the barber" referred to, is also one of the "residents of the town".

Self-reference has played a seminal role in logic and mathematics, already from the time of the greeks. From euboulides' self-referential statements, to Cantor, whose proof of the *non-denumerability* of the *real numbers* relies heavily on a numerical variant of self-reference, to Russell and his paradox, and to Gödel. In fact, Gödel proved his Incompleteness Theorem by creating, in the context of modern logic, a statement that is quite similar in spirit to that of euboulides, with one crucial difference: while euboulides states "this statement is false", Gödel's ingenious variant essentially says, in the language of arithmetic, "this statement is *unprovable*." Any consistent axiomatic theory in which one can formulate such a statement must be necessarily incomplete: for either this statement is *false*, in which case it is both false *and* provable,

contradicting the *consistency* of the axiomatic system, or *true*, in which case it is both true and unprovable, establishing its *incompleteness*.

**Set theory** The study of collections of objects united by a common property – in some cases this property can be nothing more than the fact that they are defined to be members of the same set, as for example in the arbitrarily defined set whose elements are the numbers 2, 3, 8, 134, 579. Sets were first studied by the Czech mathematician Bernard Bolzano (1781-1848), who also introduced the term *Menge* ('set') and defined the notion of a set's *cardinality*, i.e. of its "size" in a way not directly involving measurement. Thus, one can speak of two sets having the same cardinality if their elements can be put in a one-to-one correspondence – without ever needing to know via the precise number of these elements. This has the great advantage that it also works for infinite sets, where the notion of number does not apply: mathematicians don't think of "infinity" as a number. However, some seeming paradoxes, such as the fact that the *whole* and the *even* numbers can be put into a one-to-one correspondence (just by multiplying each whole number by 2, or dividing each even number by 2), thus showing a subset to have the same cardinality as the containing set, prevented Bolzano from developing the theory further. The advanced mathematical discipline of set theory was arguably born on december 7, 1873, when Georg Cantor wrote to his teacher, richard dedekind describing his proof of the *non-denumerability* of the *real* numbers (the set of the whole numbers, decimals, zero and the negative numbers), as opposed to the *denumerability* of the *rational*s (all fractions), which Cantor also proved – denumerability is defined as a one-to-one correspondence with the *natural* numbers 1, 2, 3... etc. The concept of a set is almost too primitive to merit a mathematical definition, and is practically impossible to define informally without the use of some synonym (here we used the word "collection"). It is precisely this "naturalness" of the concept in Bolzano's and Cantor's work that led to Russell's Paradox. To overcome it, and to rule out the flawed concept of "the set of all sets" it allowed for, one has to come up with bottom-up constructions and axioms for sets, as in the *Principia Mathematica* and, later, the system called "ZFC", from the names of its two creators, ernst Zermelo and Abraham Fraenkel, and the Axiom of Choice, a necessary additional axiom that allows the theory to deal with infinite sets. Set theory is considered by some the most basic branch of mathematics, as all others can be defined in terms of it. This was the gist of an over-ambitious project undertaken, from the 1930s onwards, by the group of brilliant French mathematicians writing under the pen name of "Nicolas Bourbaki".

***Tractatus Logico-Philosophicus*** Ludwig Wittgenstein wrote his seminal philosophical work during WWI, building on his pre-war notebooks and ideas on logic. It contains his solution of (in his own words) "all the problems of philosophy", dealing with the world, representation, and language. Originally called *Logische-Philosophische Abhandlung* ("Logical-Philosophical Treatise"), it was renamed for its english publication under the influence of g. e. moore, with his predilection for Latin titles. In the *Tractatus*, Wittgenstein uses many techniques and ideas from logic, especially those of Frege and Russell, as well as insights from totally different philosophical positions, mostly that of Arthur Schopenhauer. Though publication by the then totally unknown Wittgenstein was only made possible when russell accepted to write an introduction, calling the book "an important event in the philosophical world", the *Tractatus* was the cause of the two men's falling-out. Wittgenstein considered russell's – not altogether appreciative – introduction to his work to be fraught with misunderstandings and philosophical errors, while russell saw in the *Tractatus* the first signs of Wittgenstein's decline – as he saw it – into mysticism. The tight structure of the book proceeds with seven main propositions, each developed in a chapter, which are further developed in propositions arranged by a rather pedantic, and often somewhat confusing, system of numbering. The first two propositions (1 and 2) expand the positions that "the world is all that is the case", and that "what is the case" are *facts*, and *combinations* of facts. This is a departure from classical philosophy and the metaphysics of Aristotle in particular, according to which the world consists of *objects*. In the logical language of the *Tractatus*, objects do figure within states of affairs, but in complex combinations and relationships with each other, and not as elementary units. The next two propositions (3 and 4) develop mostly what has been called the *picture* theory of language, whereby a "thought is a proposition with sense." Passing here to representation and language, Wittgenstein delimits thoughts to logical propositions, but within a context and in reference to the world. This is perhaps the most subtle part of the book, and also the one which relates to Wittgenstein's idea of mathematics and logic as machines for producing *tautologies*. Propositions 5 and 6 develop the idea that "propositions are truth functions of elementary propositions", in which mathematical-symbolic notation is used to explain precisely what a truth function is. Here Wittgenstein uses logic to define propositions (and thus language and thought) as the combinations of *atomic*, or elementary propositions, combined through Boole's laws of composition. This part of



the book actually contains the first mention of what is now known as the "truth table method" for dealing with Boolean functions. The book's final clause, proposition 7, is: "What we cannot speak of, we must pass over in silence." (This and other quotes are from the d.F. Pears and B.F. mcguinness translation.) This last proposition was given two highly divergent interpretations, the extreme *positivist* one of the Vienna Circle, by which what one "cannot speak of" (logically) is, quite literally, *non-sense*, and the one that Wittgenstein and others himself later gave, which russell termed "mystical", according to which what "one cannot speak of" is the truly important. The *Tractatus* is one of the most influential and closely-studied books in Western philosophy. Its influences are legion and it may have also influenced – and certainly was vindicated by – the way in which computers and databases model the world today.



**Turing, Alan** Born in London in 1912, this great British mathematician is generally considered to be the father of computer science. Turing contributed to many areas of mathematics, but is mostly remembered for one of his earliest results in logic. While a student at Cambridge, he became fascinated by the foundations of mathematics and especially the Incompleteness Theorem of Kurt Gödel, which inspired him to study Hilbert's *Entscheidungsproblem* ("decision problem"), a question that had survived Gödel's analysis. The *Entscheidungsproblem* asks whether, given a logical system, there is an algorithm for deciding whether a proposition is

provable within the system or not. Turing's answer was a devastating "no". To reach this, he first had to define rigorously the notion of algorithm. his ingenious definition in terms of a theoretical "machine" with a *central control* and a tape for *memory, input and output*, anticipated in important ways the digital computer and has had, since then, an enormous influence on computational practice and thought. *Turing machines* – as they are now called – share with today's computers the key property of *universality*, in that a machine can carry out any computational task, provided it is supplied with an appropriate *program* for it. Two other mathematicians, Alonzo Church (later Turing's thesis advisor at Princeton) and emil Post, came up independently, and at about the same time, with algorithm formalisms that were ultimately shown

equivalent to Turing's. Yet his formalization had the greatest impact, mainly because of the extreme simplicity of its basic construction, which can, nevertheless, achieve extremely complex results. The work of Turing – as well as that of the others mentioned – on algorithms and methods for the general solvability of problems, is an obvious outgrowth of the foundational quest and thus, in a sense, its culmination. During World War II, Turing presided over the design and construction of two series of electronic computers, the "Bombe" and the "Colossus". These were used successfully – and crucially for the war effort – for breaking several German cryptographic codes, including the notoriously hard "enigma" of the German navy. After the war, Turing worked in the fledgling British computer industry, did important work in biology and founded the field of artificial intelligence by proposing what became known as the *Turing test*, a method for determining whether an artifact "can think". Always interested in sports and games – he was an accomplished long-distance runner – Turing was the first to develop ideas for a chess-playing program, making mastery in the game one of the goals towards which the designers of intelligent machines should strive. In 1952 he was prosecuted on account of his homosexuality, then a punishable offense in Britain. As an alternative to a jail sentence, he agreed to undergo an experimental "treatment" with estrogens, which probably caused the severe depression which led him to take his own life, in 1954.



**Vienna Circle** A group of philosophers and philosophically-minded scientists, who met in Vienna between 1924 and 1936. Their main aim was two-fold: to build a strong empiricist philosophy using the insights into scientific methodology garnered from recent advances in logic, mathematics and physics, and to apply

the methodology of the physical sciences to the social. The scientifically-trained philosopher of science Moritz Schlick is generally recognized to be the group's leader. Some of the most prominent members were: the mathematicians Hans Hahn, Olga Hahn-Neurath, Gustav Bergmann, Karl Menger, and Kurt Gödel for a short period of time; the physicist Philipp Frank; the social scientist Otto Neurath and the philosophers Viktor Kraft and Rudolf Carnap. The group met informally Thursday evenings at Vienna's "Café Central", but was later constituted as a society with public meetings. Despite the group's informal nature,

the members had a common core of philosophical beliefs, expressed in a sort of manifesto, titled "The Scientific Conception of the World". The members of the Circle declared that the work of Frege, Russell and Einstein provided their first inspiration, while the *Tractatus Logico-Philosophicus* of Ludwig Wittgenstein functioned as their direct model. The philosophies of *logical positivism* and *logical empiricism*, expressing the worldview of the members of the Circle, state that knowledge comes from experience – and thus, basically, from scientific observation and experiment – developed into theory through logical analysis and synthesis. Still, following the *Tractatus*, members of the Vienna Circle held that logic and mathematics only deal in *tautologies*, and thus do not provide knowledge as such, but only one of the tools for the elaboration of empirical knowledge. According to the worldview of the Circle, statements that cannot be reduced to experience (such as theological or ethical pronouncements) cannot be right or wrong, as they are – quite literally – *non-sense*, having no meaning. The most extreme version of this tenet, due to Carnap, actually required that for a statement to be meaningful, its truth or falsity must be verifiable by an algorithm reducing it to observable truths – a new incarnation of Leibniz's "*calculus*". Carnap later tried to reconcile this view with the Incompleteness Theorem. Though the Vienna Circle, in its original form, was dissolved in 1936, after Schlick's murder by a paranoid ex-student and Nazi sympathizer, its spirit continued to live on. Most of its members managed to flee Austria and emigrate to England and the United States, where they had a major influence on the development of post-war philosophy.



**Von Neumann, John** Born in Budapest in 1903 ("John" is the anglicized form of the Hungarian "János"), von Neumann showed very early signs of unusual intellectual prowess, being able to do mental division of 8-digit numbers and converse in ancient Greek by the age of six. He studied mathematics in Budapest, obtaining a Ph.D. at 22, meanwhile also working towards a degree in chemical engineering at the renowned Technical University of Zürich, to please his father.

He rapidly became the star mathematician of his generation, legendary for his penetrating and rapid-fire mathematical genius. Upon attending the lecture where Gödel announced the first Incompleteness Theorem,

von Neumann was the first to realize the result's import, and did indeed proclaim "it's all over" after the talk. But he made crucial suggestions to Gödel right after it, and went on to prove the second incompleteness Theorem – which however Gödel himself had also proven independently in the meantime. Von Neumann never worked on the foundations of mathematics again. Possessing a wide-ranging mathematical genius, he made contributions to many different branches, he has been called "the last of the great mathematicians", having made great contributions to many different branches of mathematics, among them set theory, operator algebras, ergodic theory and statistics. He also did important work in quantum theory, fluid mechanics and mathematical economics, being the co-founder (with economist oscar morgenstern) of the field of *game theory*. During WWII, he was one of the brains behind the atomic bomb, and after it headed the U.S. government committee in charge of the construction of the hydrogen bomb. Perhaps most important of all his work, however, was his contribution to the creation of computers. While he was working as a consultant in the design of one of the first electronic computers, in 1946, and influenced by Alan Turing's ideas, von Neumann developed an array of fundamental design principles, postulating, among others, a *central processing unit* and separate *memory* devices where both *data* and *programs* are both stored. Practically all subsequent computer designs have been based on this basic model, now known as the *von Neumann architecture*. Von Neumann went on to become one of the first great computer scientists, especially excelling in what now would be called *scientific computing*, i.e. the use of computers for scientific research. He died of cancer – possibly the result of his attendance of thermonuclear tests – in 1957.



**Whitehead, Alfred North** english mathematician and philosopher. Born in 1861, he studied mathematics at Cambridge, where he also taught for many decades. In 1891 he married evelyn Wade, an irish woman much younger than himself. Before his intense, decade-long collaboration with Bertrand Russell on the *Principia Mathematica*, Whitehead published his book *Universal Algebra*, an attempt to study the types of symbolic reasoning

in various algebraic systems from a very modern – for its time – formal viewpoint. After russell's abandonment of the *Principia*, in 1913, Whitehead

tried to write a fourth volume, on geometry, but never completed it. The two men had very little interaction after the publication of the *Principia*, and Whitehead did not contribute to the 1925, second edition of the book, having moved on to mathematical physics and later philosophy. He died in 1947.



**Wittgenstein, Ludwig** Wittgenstein is considered by many to be the greatest philosopher of the 20<sup>th</sup> century. He was one of the eight children of industrialist Karl Wittgenstein, one of Austria's wealthiest and most powerful men, and a great patron of the arts. Of his four brothers, three committed suicide in early manhood, while the fourth, Paul, went on to become a renowned concert pianist. After two years of engineering studies, Wittgenstein

developed a strong interest in logic and the foundations of mathematics. He went to see Frege, who suggested that he go to Cambridge to study with Russell, a piece of advice Wittgenstein followed. The association deeply influenced both men, but probably the teacher more than the student. During his service with the Austro-hungarian army in WWI, Wittgenstein won several medals for his valour, his citations underlining his "sang-froid under fire". He was eventually captured by the enemy and completed his magnum opus, the *Tractatus Logico-Philosophicus*, in an Italian prisoners' camp. After the war he donated the huge fortune left to him by his father to his three sisters and, having, as he believed and declared, "solved all the problems of philosophy" with the *Tractatus*, he worked as a gardener, architect, and eventually as a teacher in a small village in Lower Austria. In 1929, possibly inspired by interactions with members of the Vienna Circle, as well as attending a lecture on the philosophy of mathematics by Luitzen Brouwer, on intuitionism, Wittgenstein returned to Cambridge and philosophy. He retracted his earlier work as dogmatic and went on to create a new, extremely influential philosophical stance often referred to as "the late Wittgenstein". Unlike the ideas in the *Tractatus*, Wittgenstein did not attempt to put his later philosophy in a systematic treatise, but presented them in a series of more or less independent remarks. Many of these he saw as forming a book, which was posthumously published as *Philosophical Investigations* – this, as well as a few books based on his notebooks, or transcripts of lectures or discussions, are all that we have of his later thought. This is a

philosophical position of an extreme anti-dogmatic nature, focusing on *language* and *psychology* (what we now call *cognitive psychology*), instead of logic and objective truth, and on fuzzy concepts such as "family resemblance" and "language games" instead of clear definitions and propositions. In this later phase, Wittgenstein's thinking is characterized by a vicious criticism of philosophy as it had been practiced until then, by others but also himself – it was for this criticism more than anything else, that Russell was dismissive of his later work, referring to Wittgenstein's decision to "become a mystic". Most of his negative criticism of mathematics – which he increasingly came to view as a purely practical activity, a craft legitimized only by its use in application – is contained in transcriptions of his lecture notes at Cambridge. Of particular interest is the dialogue with one of the attendees at these lectures, Alan Turing, who strongly disagreed with his ideas on mathematics. Wittgenstein died in 1951.

## Bibliography

In preparation for *Logicomix* we read many books – in addition to those we had read earlier, before the idea for the project was even born – and consulted many more, and even more articles. Of all these, we mention here very few, chosen either for the wealth of the information they contain, for their astuteness, profundity and/or synthetic ability. Clearly, this list represents a personal choice, and nothing more: these are the books that we most liked and found most useful.

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