Introduction to Logic [296.617]

2nd Midterm Take - home test

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1. For each of the following arguments, say if it is valid or not and, in each case, please justify your answer. [20 %]

   a)

(1)	Every Italian is mortal.
(2)	Francesco is mortal.
(3)	Francesco is Italian.

   b)

(1)	If the Blue Whale is the biggest animal on Earth, then there is a Blue Whale named "Dave".
(2)	The Blue Whale is the biggest animal on Earth
(3)	There is a Blue Whale named "Dave".

   c)

(1)	$A \to \neg B$
(2)	A
(3)	$\neg B \vee B$

2. For each of the following sentences, use the truth-table method to find out whether it is contingent, a tautology, or a logical falsehood. [15 %]
   a) $((\varphi \to \psi) \wedge (\psi \to \varphi)) \leftrightarrow (\varphi \wedge \psi)$
   b) $(\varphi \vee \psi) \leftrightarrow (\neg\varphi \to \psi)$
   c) $\neg((((\varphi \vee \psi) \to \chi) \vee \varphi) \vee \psi)$

3. Translate the following sentences: [20 %]
   a) **From English to the language of propositional logic**: If Joey knows something and Rachel knows something, then they know the same thing.
   b) **From the language of first-order logic to English (please make use of the translation key provided)**: $((S(x,y) \to P(x,y)) \wedge \neg P(x,y)) \to \neg S(x,y)$
      - $S(x,y)$:= x studied y;
      - $P(x,y)$:= x passed y;
   c) **From English to the language of first-order logic**: If every man is Greek, then every man is mortal.
   d) **From the language of first-order logic to English (again, please make use of the translation key provided)**: $\neg\exists x\forall y[N(x) \wedge N(y) \wedge M(x,y)]$, where
      - $N(x)$:= x is a natural number;
      - $M(x,y)$:= x is bigger than y.

4. Consider the following sentence: [15 %]

$$\forall x\forall y[R(x,y) \vee R(y,x)], \text{with } x \neq y.$$

a) Is it satisfiable? (provide an example)

b) Is it valid? (if not, provide a counterexample)

5. Prove the following theorem using Natural Deduction Rules for propositional logic (primitive rules only): [15 %]

$$\vdash_{ND} (\varphi \to \psi) \lor (\psi \to \varphi).$$

6. Prove the following sentence using Natural Deduction Rules for first-order logic (primitive rules only): [15 %]

   a) $\vdash_{ND} \forall x[A(x)] \to \forall x[A(x) \lor B(x)]$.

Solutions

1. Arguments:

   a) Invalid. There could be the case where Francesco is mortal but not Italian.

   b) Valid. The first premise is a conditional, while the second premise states that the antecedent is true. Then it cannot be that both premises are true and the conclusion is false.

   c) Valid.

2. Semantic for propositional logic:

   a) Contingent

φ	ψ	$\varphi \to \psi$	$\psi \to \varphi$	$(\varphi \to \psi) \land (\psi \to \varphi)$	$\varphi \land \psi$	$((\varphi \to \psi) \land (\psi \to \varphi)) \leftrightarrow (\varphi \land \psi)$
1	1	1	1	1	1	1
1	0	0	1	0	0	1
0	1	1	0	0	0	1
0	0	1	1	1	0	0

   b) Tautology

φ	ψ	$\varphi \lor \psi$	$\neg\varphi$	$\neg\varphi \to \psi$	$(\varphi \lor \psi) \leftrightarrow (\neg\varphi \to \psi)$
1	1	1	0	1	1
1	0	1	0	1	1
0	1	1	1	1	1
0	0	0	1	0	1

   c) Contradiction

φ	ψ	χ	$\varphi \lor \psi$	$(\varphi \lor \psi) \to \chi$	$(\varphi \lor \psi) \to \chi) \lor \varphi$	$((((\varphi \lor \psi) \to \chi) \lor \varphi) \lor \psi)$	$\neg A$
1	1	1	1	1	1	1	0
1	1	0	1	0	1	1	0
1	0	1	1	1	1	1	0
1	0	0	1	0	1	1	0
0	1	1	1	1	1	1	0
0	1	0	1	0	0	1	0
0	0	1	0	1	1	1	0
0	0	0	0	1	1	1	0

3. Translations:

   a) $(K(J, x) \land K(R, y)) \to x = y$;

      i. $K(x, y) :=$ x knows y;

   b) If x studied y then x passed y. But x didn't passed y. Then x didn't studied y.

   c) $\forall x[(M(x) \land G(x))] \to \forall x[Mo(x)]$.

i. $M :=$ x is a man;

ii. $G :=$ x is Greek;

iii. $Mo :=$ x is mortal;

d) There is no natural number bigger than all. / There are infinite natural numbers.

4. First order semantics:

   a) $\forall x \forall y[R(x,y) \vee R(y,x)]$ is satisfiable by the following structure $\langle \mathfrak{M}, s\rangle$:

      - Domain: $\mathcal{D} = \{\mathbb{N}\}$;
      - $R^{\mathfrak{M}}(n,m) := n \leq m$, for all $n, m \in \mathcal{D}$;
      - $s(x) = n \in \mathbb{N}, s(y) = m \in \mathbb{N}$.

   b) On the other hand, $\forall x \forall y[R(x,y) \vee R(y,x)]$ is not valid. Consider as a counterexample the following structure:

      - Domain: $\mathcal{D} = \{2, 3\}$;
      - $R^{\mathfrak{M}}(n,m) := n$ is half than m, for all $n, m \in \mathcal{D}$;
      - $s(x) = 3, s(y) = 2$.

5. Natural Deduction for propositional logic:

$$\dfrac{\dfrac{\dfrac{\dfrac{[\varphi \to \psi]^1}{(\varphi \to \psi) \vee (\psi \to \varphi)}\langle \vee\mathrm{I}\rangle \quad [\neg((\varphi \to \psi) \vee (\psi \to \varphi))]^2}{\bot}\langle \neg\mathrm{I}\rangle \quad \dfrac{\dfrac{[\psi \to \varphi]^1}{(\varphi \to \psi) \vee (\psi \to \varphi)}\langle \vee\mathrm{I}\rangle \quad [\neg((\varphi \to \psi) \vee (\psi \to \varphi))]^2}{\bot}\langle \neg\mathrm{I}\rangle \quad \dfrac{[\varphi \to \psi]^1}{(\varphi \to \psi) \vee (\psi \to \varphi)}\langle \vee\mathrm{I}\rangle}{\bot}\langle \vee\mathrm{E}\rangle,1}{(\varphi \to \psi) \vee (\psi \to \varphi)}\langle \mathrm{CR}\rangle,2$$

6. Natural Deduction for first order logic:

   a)

$$\dfrac{\dfrac{\dfrac{\dfrac{[\forall x[A(x)]]^1}{A(y)}\langle \forall\mathrm{E}\rangle}{A(y) \vee B(y)}\langle \vee\mathrm{I}\rangle}{\forall x[A(x) \vee B(x)]}\langle \forall\mathrm{I}\rangle}{\forall x[A(x)] \to \forall x[A(x) \vee B(x)]}\langle \to\mathrm{I}\rangle$$